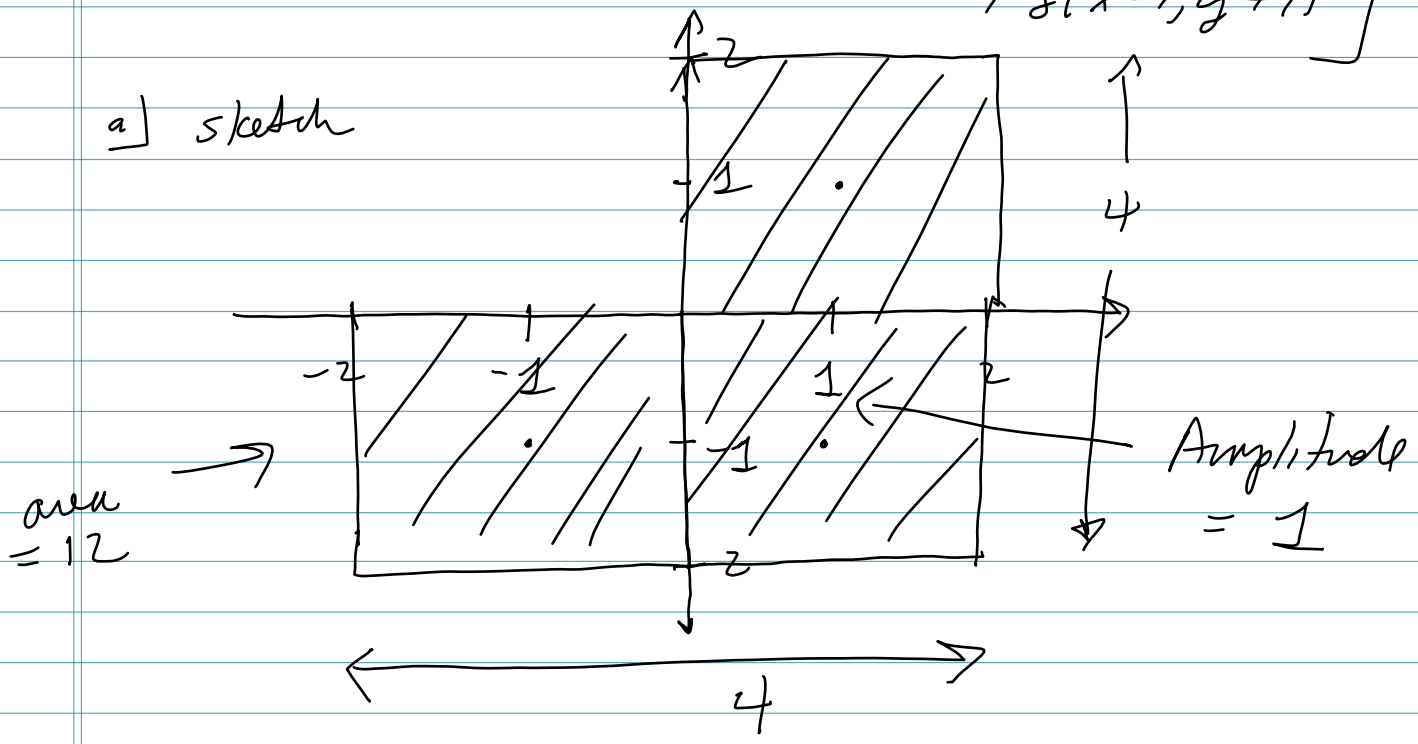
BE280A15

Problem 1

$$f(x,y) = \text{rect}\left(\frac{x}{2}, \frac{y}{2}\right) ** \left[\delta(x-1, y-1) + \delta(x+1, y+1) + \delta(x-1, y+1)\right]$$

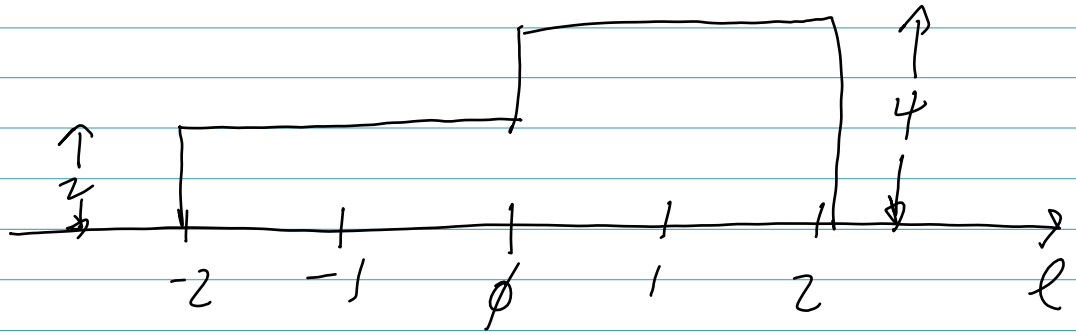
a] sketch



Note: we can write this object as

$$f(x,y) = \text{rect}\left(\frac{x}{4}, \frac{y}{4}\right) - \text{rect}\left(\frac{x+1}{2}, \frac{y-1}{2}\right)$$

b] Projection at zero degrees



area = 12

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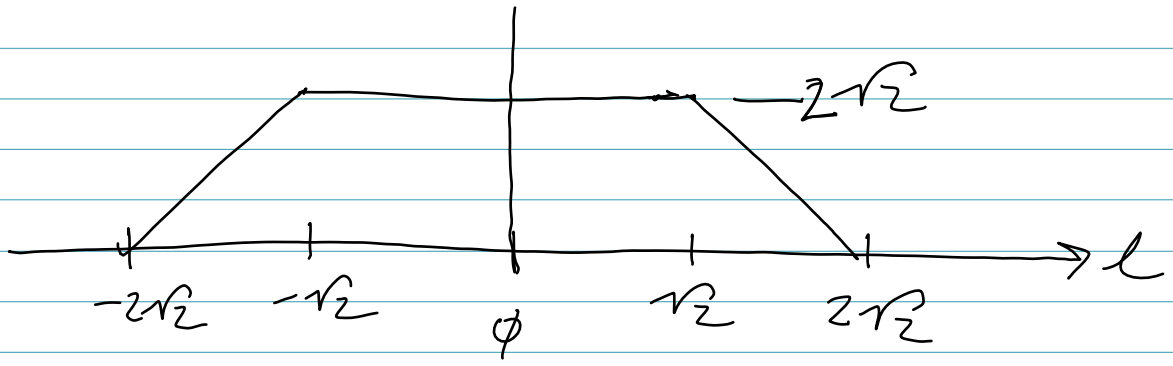
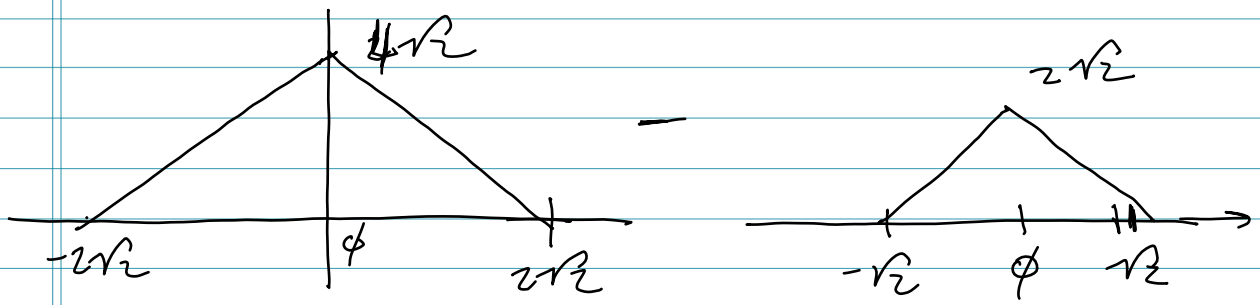
c) Projections at 45 degrees

area =



note we can also see that this is just the difference of the projections of the two rect functions



d) Fourier Transform

$$F(K_x, K_y) = 16\,\mathrm{sinc}(4K_x, 4K_y) - \mathcal{F}\left(\mathrm{rect}\left(\frac{x+1}{2}, \frac{y-1}{2}\right)\right)$$

$$= 16\,\mathrm{sinc}(4K_x, 4K_y) - \mathcal{F}\left(\mathrm{rect}\left(\frac{x}{2}, \frac{y}{2}\right) * \delta(x+1, y-1)\right)$$

$$= 16\,\mathrm{sinc}(4K_x, 4K_y) - 4\,\mathrm{sinc}(2K_x, 2K_y) \cdot e^{j2\pi K_x} \cdot e^{-j2\pi K_y}$$

e) at ϕ degrees

$K_x = K \quad K_y = \phi$

$$F(K, 0) = 16\,\mathrm{sinc}(4K) - 4\,\mathrm{sinc}(2K)\, e^{j2\pi K}$$

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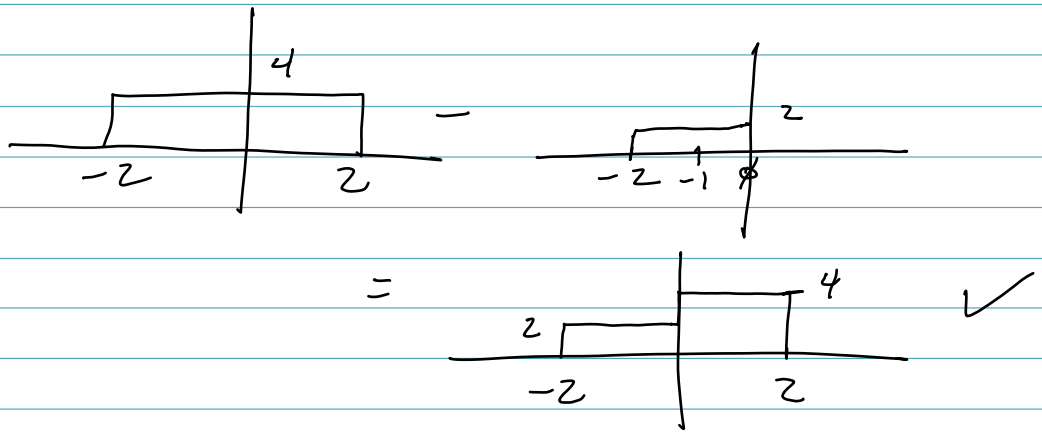
e] continued

$$\mathcal{F}^{-1}\left(F(k,0)\right) = 4\,\mathrm{rect}\left(\frac{\ell}{4}\right) - 2\,\mathrm{rect}\left(\frac{\ell}{2}\right) * \delta(\ell+1)$$

$$= 4\,\mathrm{rect}\left(\frac{\ell}{4}\right) - 2\,\mathrm{rect}\left(\frac{\ell+1}{2}\right)$$



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f] at 45 degrees $k_x = k_y = \frac{\sqrt{2}}{2}k$

$$F\left(\frac{\sqrt{2}}{2}k, \frac{\sqrt{2}}{2}k\right) = 16\,\mathrm{sinc}\left(2\sqrt{2}k, 2\sqrt{2}k\right) - 4\,\mathrm{sinc}\left(\sqrt{2}k, \sqrt{2}k\right) \underbrace{e^{j2\pi\frac{\sqrt{2}}{2}k} e^{-j2\pi\frac{\sqrt{2}}{2}k}}_{=1}$$

$$= 16\,\mathrm{sinc}^2\left(2\sqrt{2}k\right) - 4\,\mathrm{sinc}^2\left(\sqrt{2}k\right)$$

$$\mathcal{F}^{-1}\left(F\left(\frac{\sqrt{2}}{2}k, \frac{\sqrt{2}}{2}k\right)\right) = 16\cdot\frac{1}{2\sqrt{2}}\Lambda\left(\frac{1}{2\sqrt{2}}\ell\right) - \frac{4}{\sqrt{2}}\Lambda\left(\frac{1}{\sqrt{2}}\ell\right)$$

$$= 4\sqrt{2}\,\Lambda\left(\frac{\ell}{2\sqrt{2}}\right) - 2\sqrt{2}\,\Lambda\left(\frac{\ell}{\sqrt{2}}\right)$$

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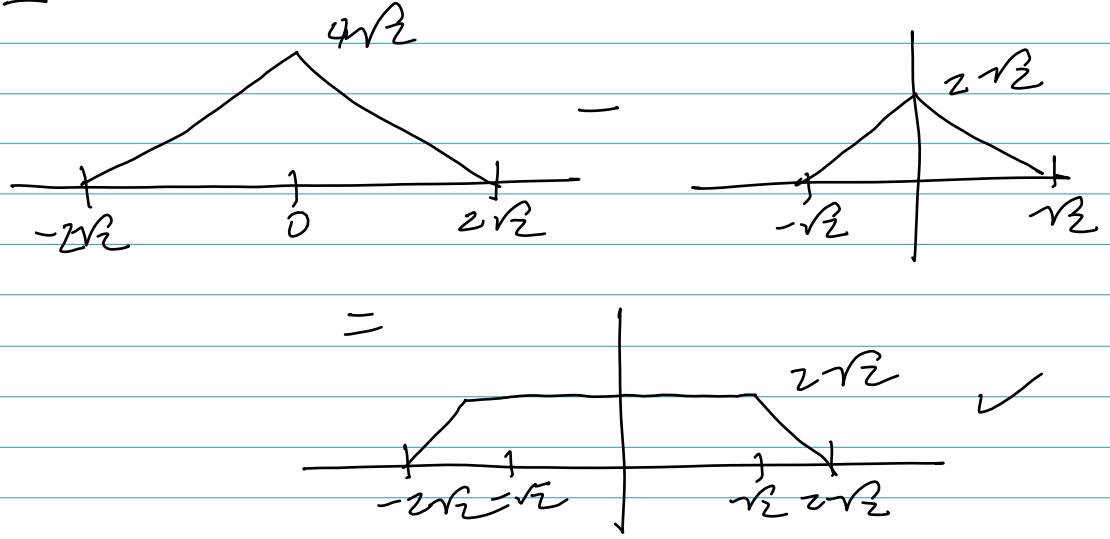
f] cont'd



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g] crude backprojection

2	4
2	4

+

2	2
4	4

=

4	6
6	8

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Projection at 30°

$$k_x = k\cos 30 = \sqrt{3}/2\,k$$
$$k_y = k\sin 30 = \frac{1}{2}k$$

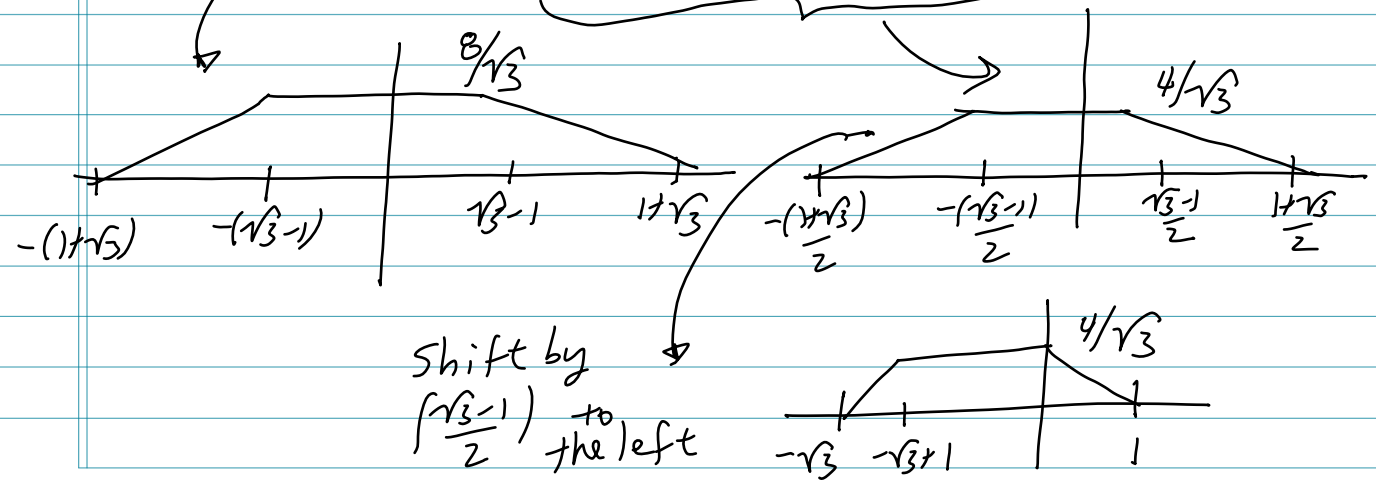
$$G(k) = F\left(k\sqrt{3}/2, \frac{k}{2}\right) =$$

$$16\,\mathrm{sinc}\left(4\frac{\sqrt{3}}{2}k, 4\cdot\frac{k}{2}\right) - 4\,\mathrm{sinc}\left(2\frac{\sqrt{3}}{2}k, 2\cdot\frac{k}{2}\right) \cdot e^{+j2\pi\sqrt{3}/2\,k}\cdot e^{-j2\pi\frac{k}{2}}$$

$$= 16\,\mathrm{sinc}(2\sqrt{3}k)\,\mathrm{sinc}(2k) - 4\,\mathrm{sinc}(\sqrt{3}k)\,\mathrm{sinc}(k) \cdot e^{j2\pi\sqrt{3}/2\,k}\,e^{-j2\pi\frac{k}{2}}$$

$$g(\ell) = \mathcal{F}^{-1}(G(k)) = 16\frac{1}{2\sqrt{3}}\cdot\frac{1}{2}\,\mathrm{rect}\left(\frac{\ell}{2\sqrt{3}}\right) * \mathrm{rect}\left(\frac{\ell}{2}\right) - 4\frac{1}{\sqrt{3}}\,\mathrm{rect}\left(\frac{\ell}{\sqrt{3}}\right)\mathrm{rect}(\ell) * \delta(\ell+\sqrt{3}/2) * \delta(\ell-1/2)$$

$$= \underbrace{4\frac{1}{\sqrt{3}}\,\mathrm{rect}\left(\frac{\ell}{2\sqrt{3}}\right) * \mathrm{rect}(\ell/2)}_{} - \underbrace{\frac{4}{\sqrt{3}}\,\mathrm{rect}\left(\frac{\ell}{\sqrt{3}}\right)\mathrm{rect}(\ell) * \delta(\ell+\sqrt{3}/2-1/2)}_{}$$



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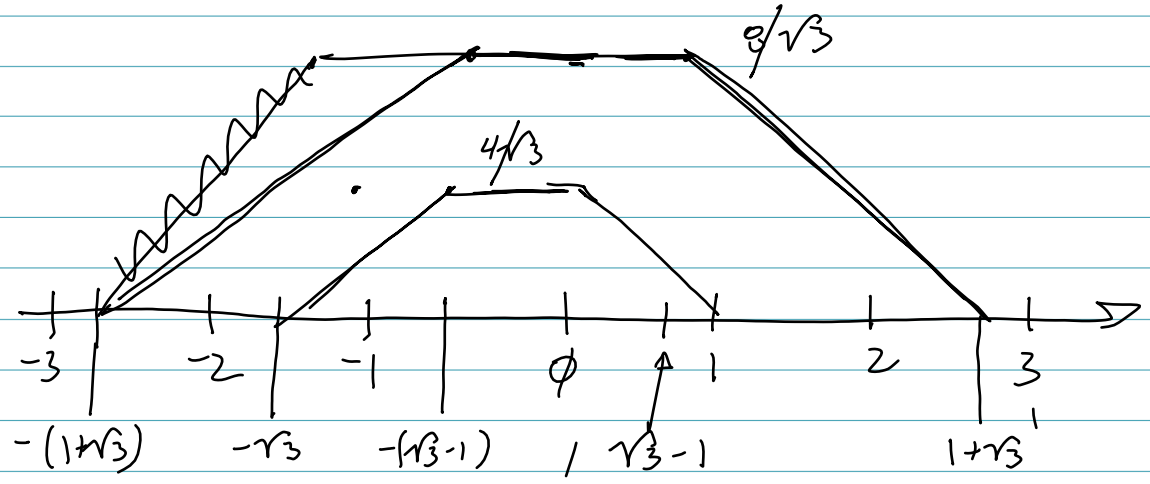
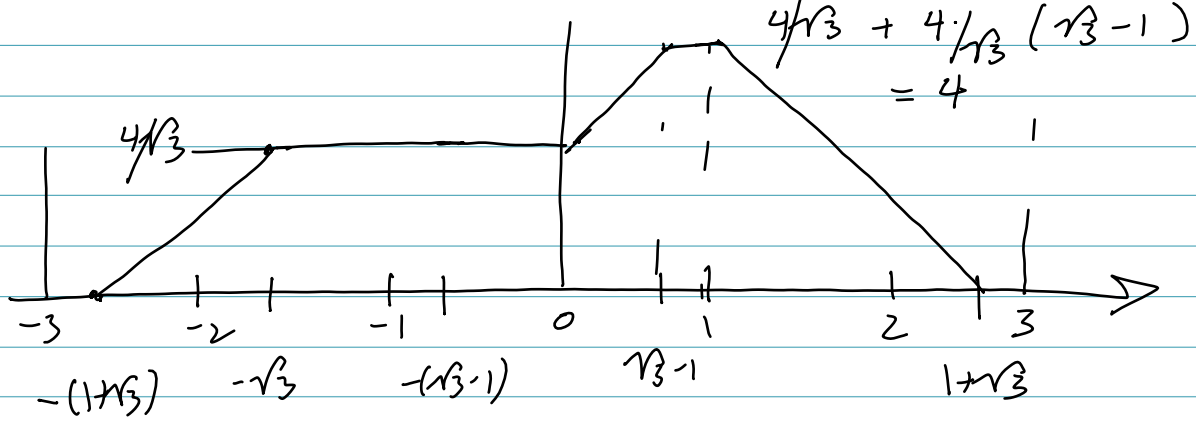
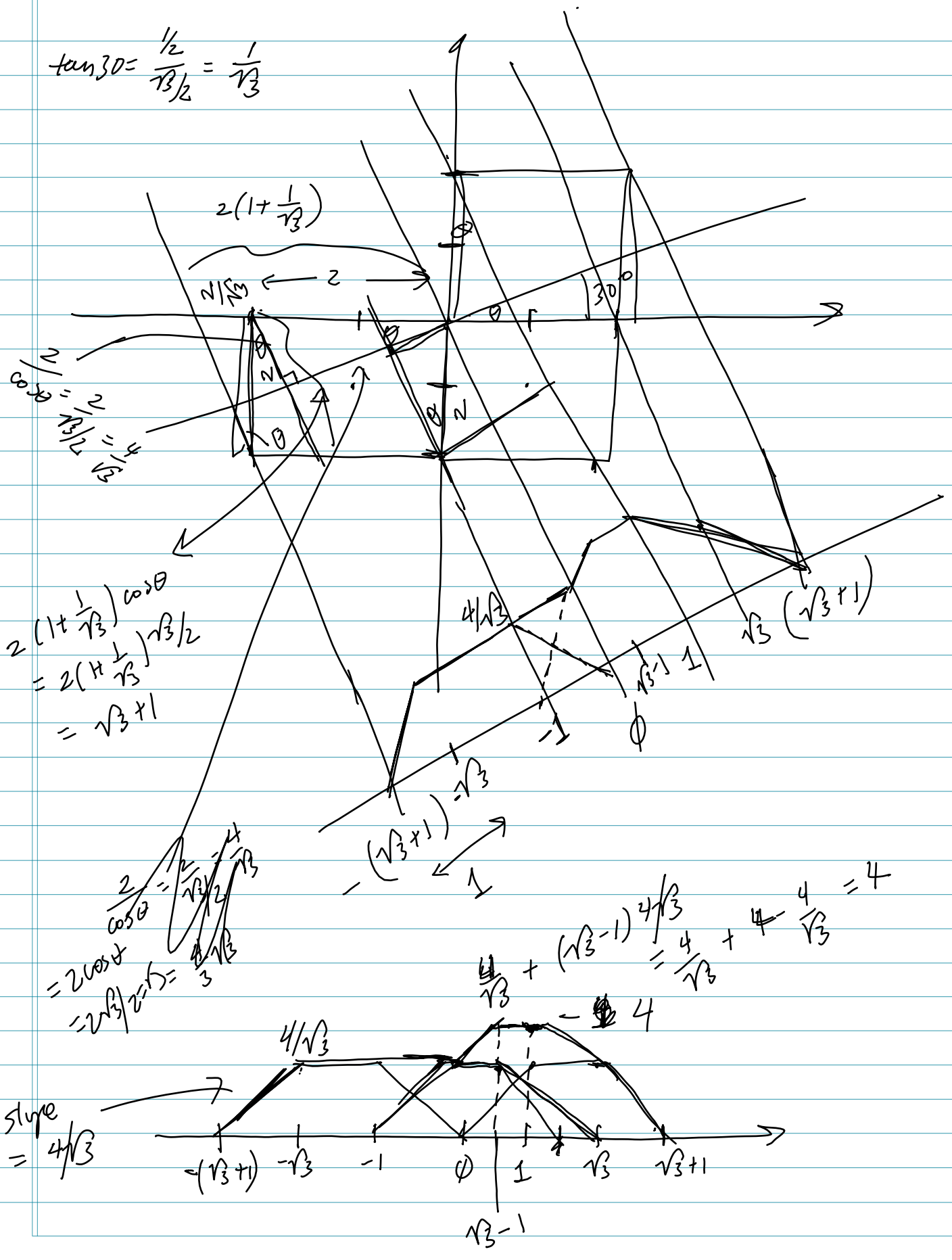




$\tan 30 = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$



BE280A/5 Exam

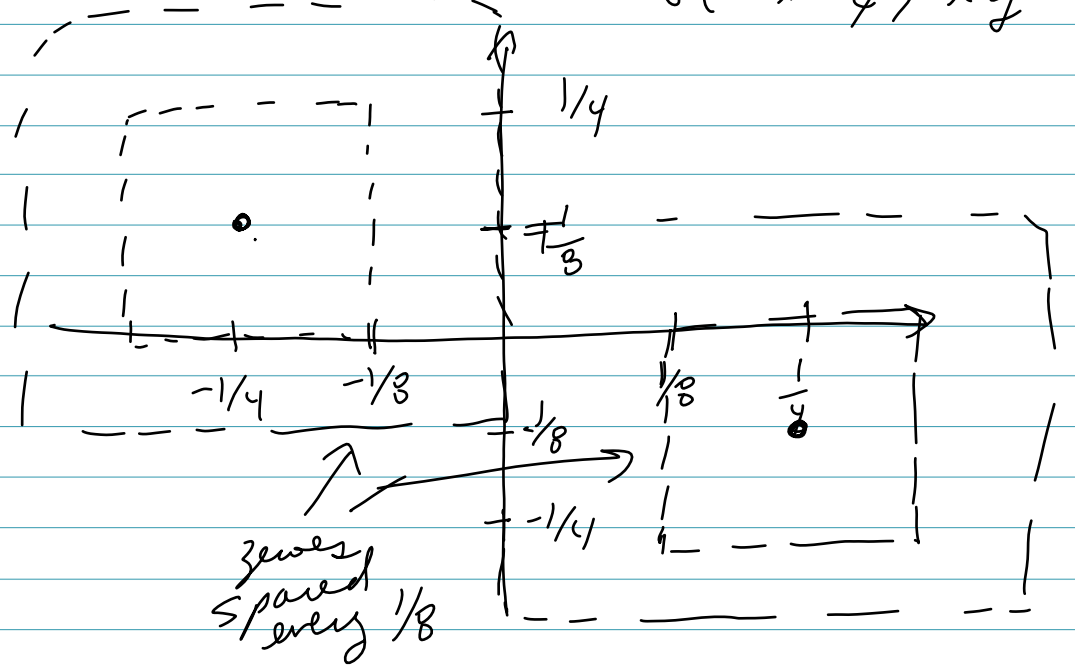
Problem 2

a] Expression for object

$$g(x,y) = \text{rect}\left(\frac{x}{8}, \frac{y}{8}\right) \cos\left(2\pi\left(\frac{1}{4}x - \frac{1}{8}y\right)\right)$$

b] $G(k_x, k_y) = 64\,\text{sinc}(8k_x, 8k_y)$

$$* \frac{1}{2}\left[\delta\left(k_x - \tfrac{1}{4}, k_y + \tfrac{1}{8}\right) + \delta\left(k_x + \tfrac{1}{4}, k_y - \tfrac{1}{8}\right)\right]$$



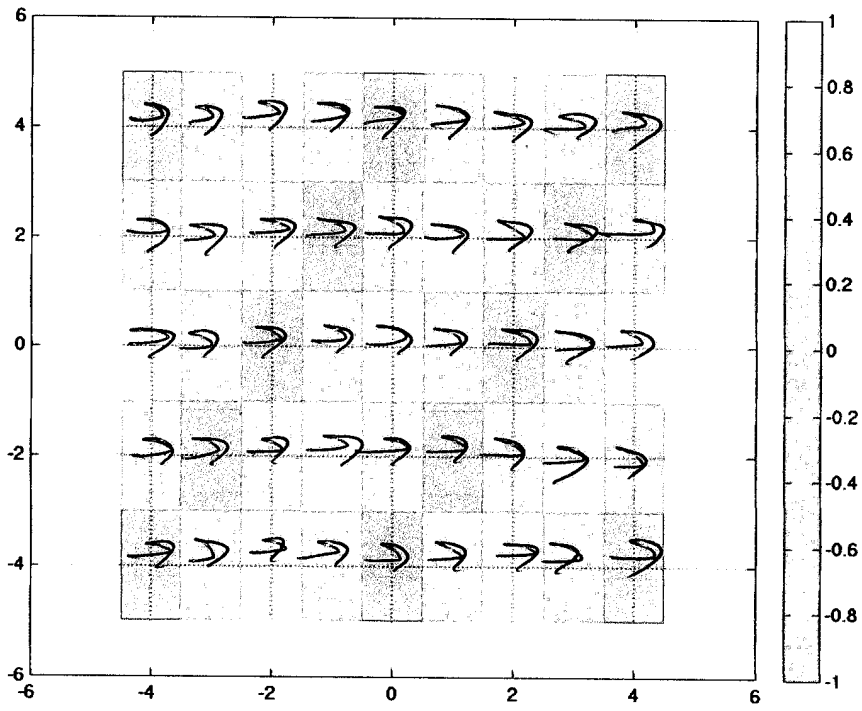
c] $FOV_x = FOV_y = 8\text{ cm} \rightarrow \Delta k_x = \Delta k_y = \frac{1}{8}\text{ cm}^{-1}$

d] $k_{x,max} = \frac{W_{kx}}{2} = \frac{1}{2\delta_x} = \frac{1}{2}\text{ cm}^{-1}$

$k_{y,max} = \frac{W_{ky}}{2} = \frac{1}{2\delta_y} = \frac{1}{4}\text{ cm}^{-1}$

e)

row sum



-1

ϕ

1

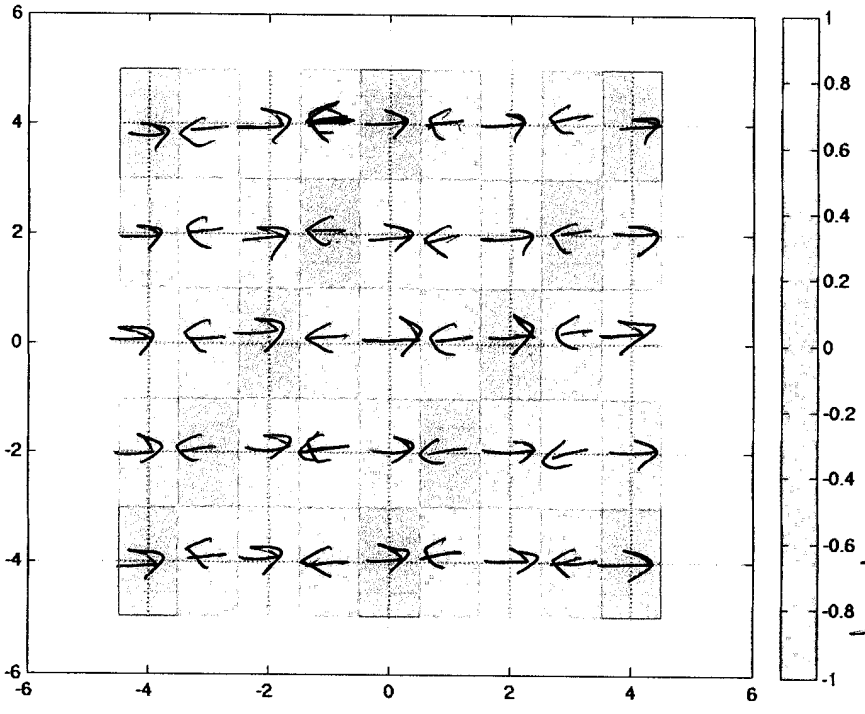
ϕ

-1

sum $= -1$

f)

row sum



-1

ϕ

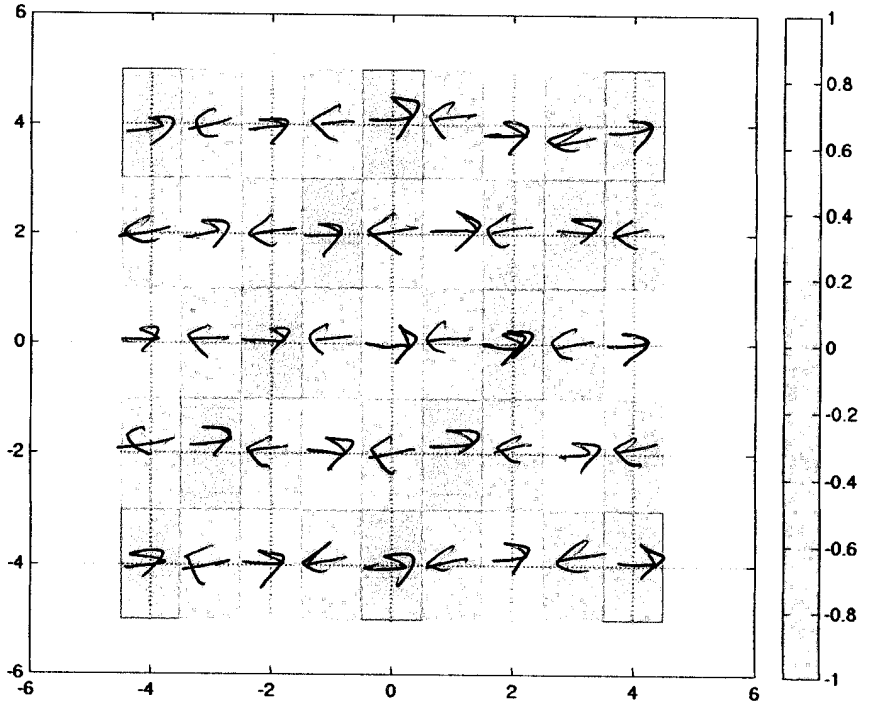
1

ϕ

-1

sum $= -1$

a)



row sum

-1

ϕ

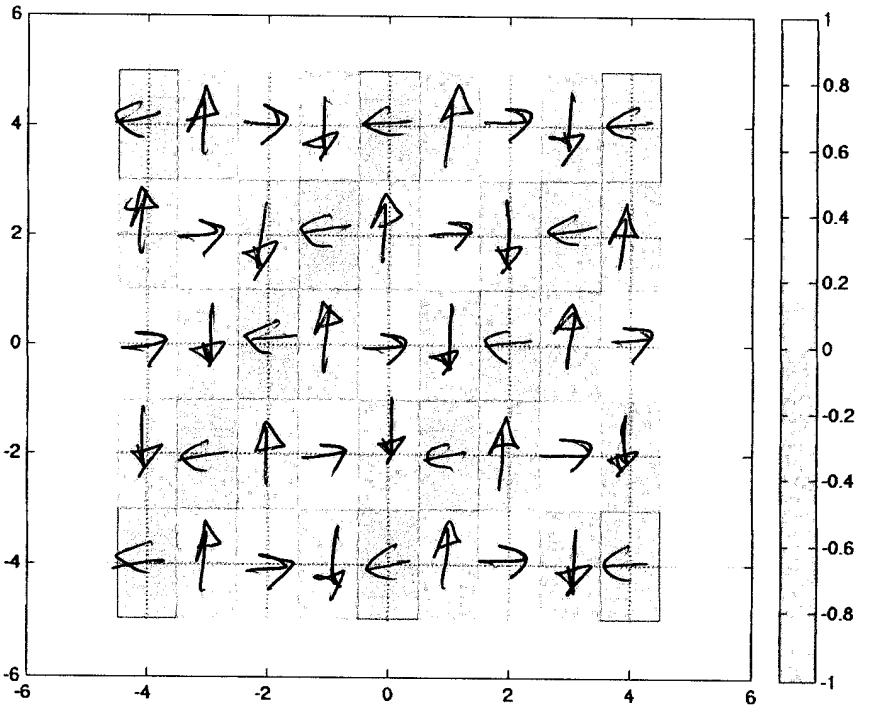
1

ϕ

-1

-1

b)



row sum

5

~~SAME~~ 4

4

5

4

5

23

$K_x = \frac{1}{4} \quad K_y = -\frac{1}{8}$