Neuroscience 200C

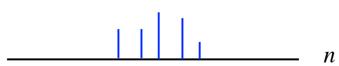
Spring Quarter 2005
Imaging/MRI Lecture

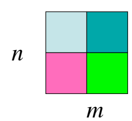
Topics

1. Representing Images
2. 2D Fourier Transform
3. MRI Basics
4. MRI Applications
5. fMRI

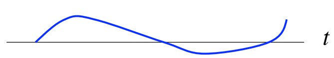
Signals and Images

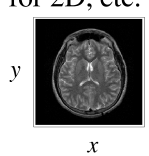
Discrete-time/space signal/image: continuous valued function with a discrete time/space index, denoted as *s[n]* for 1D, *s[m,n]* for 2D , etc.





Continuous-time/space signal/image: continuous valued function with a continuous time/space index, denoted as *s(t)* or *s(x)* for 1D, *s(x,y)* for 2D, etc.





2D Image

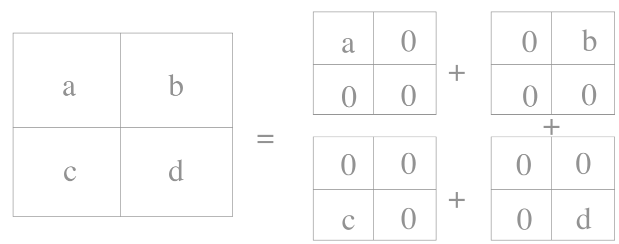


Image Decomposition

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + d\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{aligned} g[m,n] &= a\delta[m,n] + b\delta[m,n-1] + c\delta[m-1,n] + d\delta[m-1,n-1] \\ &= \sum_{k=0}^{1}\sum_{l=0}^{1} g[k,l]\delta[m-k,n-l] \\ &= \sum_{k=0}^{1}\sum_{l=0}^{1} c_{k,l} b_{k,l}[m,n] \end{aligned}$$

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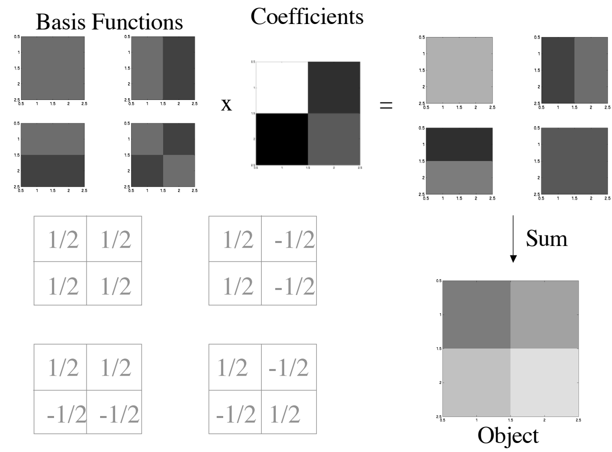

$$\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \quad \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & -1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & 1/2 \\ -1/2 & -1/2 \end{bmatrix} \quad \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

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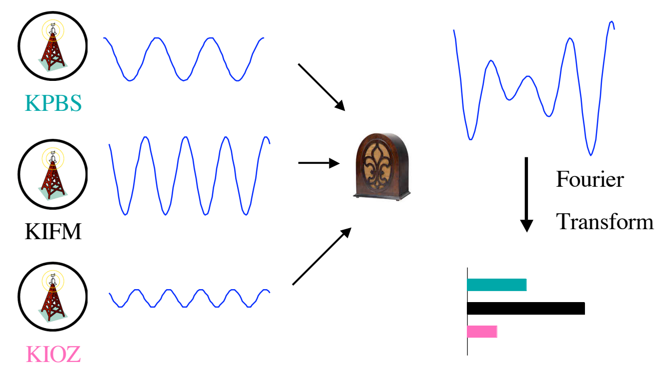
1D Fourier Transform



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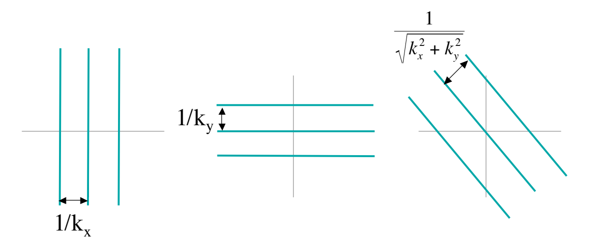
2D Plane Waves

$$e^{j2\pi(k_x x + k_y y)} = \cos\left(2\pi(k_x x + k_y y)\right) + j\sin\left(2\pi(k_x x + k_y y)\right)$$



$\cos(2\pi k_x x)$ $\cos(2\pi k_y y)$ $\cos(2\pi k_x x + 2\pi k_y y)$

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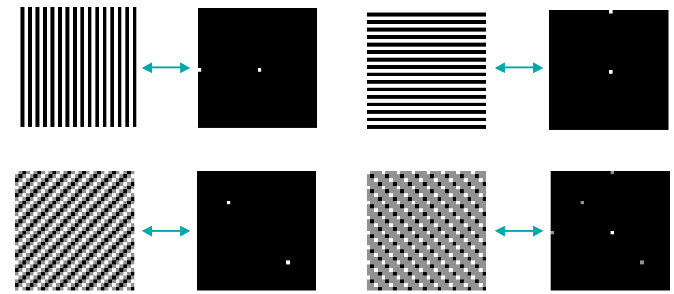
2D Fourier Transform



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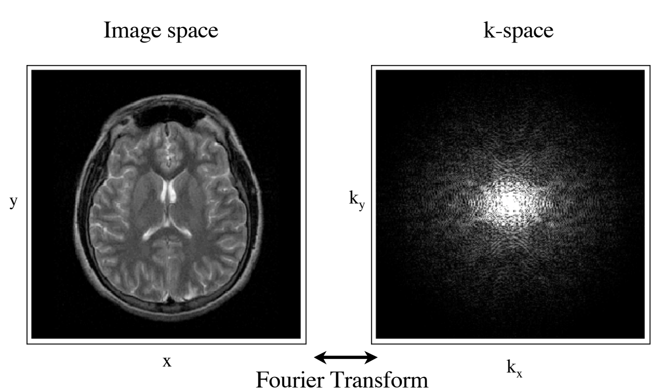
k-space



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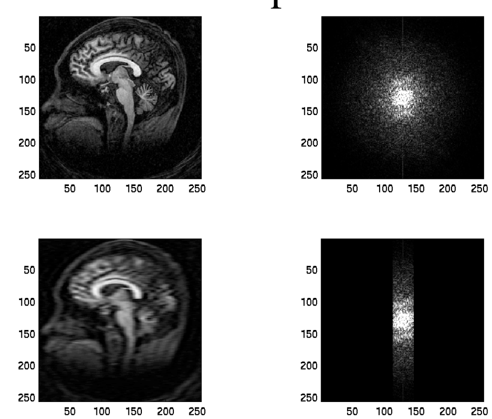
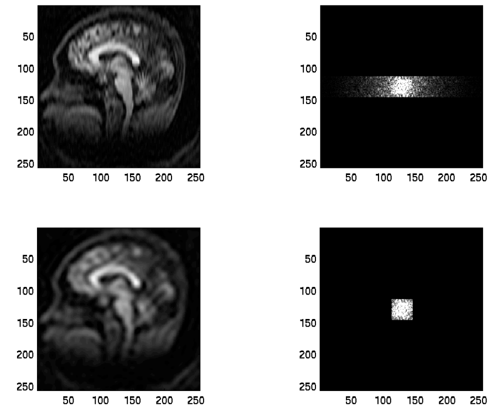
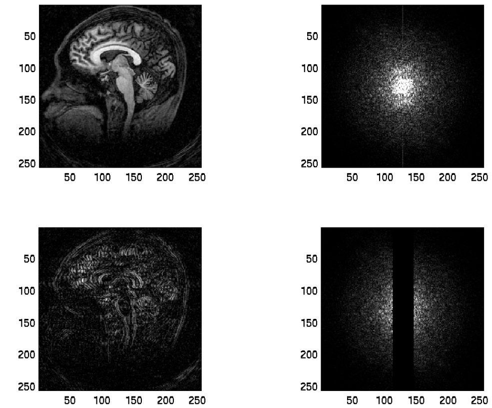
Examples



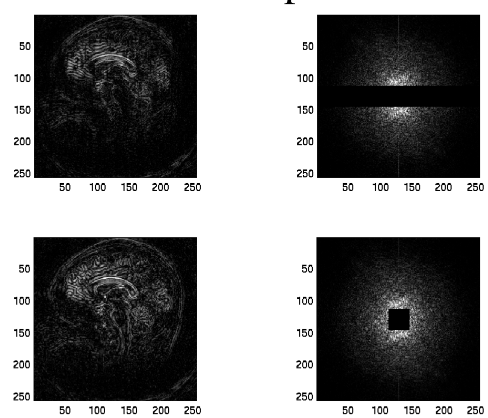
Examples



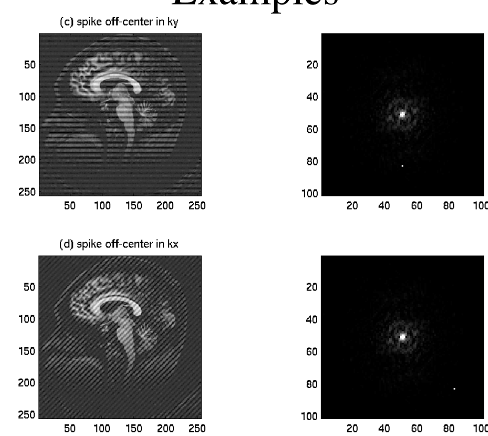
Examples



Examples



Examples



2D Fourier Transform

Fourier Transform

$$G(k_x,k_y) = F\left[g(x,y)\right] = \left\langle e^{j2\pi(k_x x+k_y y)}, g\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi(k_x x+k_y y)}dxdy$$

Inverse Fourier Transform

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(k_x,k_y)e^{j2\pi(k_x x+k_y y)}dk_x dk_y$$

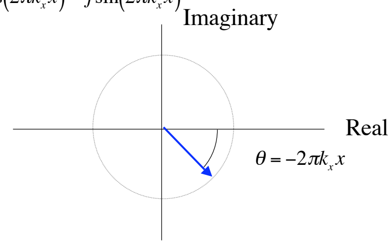
Phasor Diagram

Recall that a complex number has the form

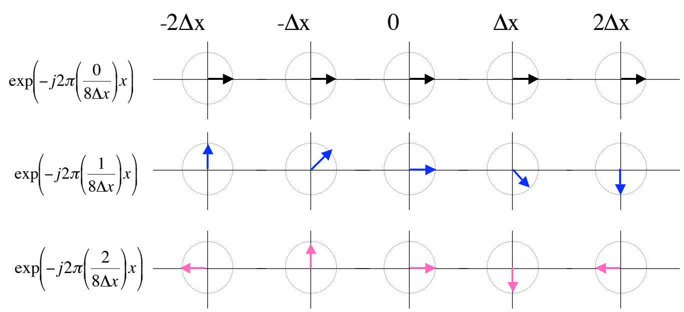
$$z = a + jb = |z|\exp(j\theta) = |z|(\cos\theta + j\sin\theta)$$

where $|z| = \sqrt{a^2+b^2}$ and $\theta = \tan^{-1}(b/a)$

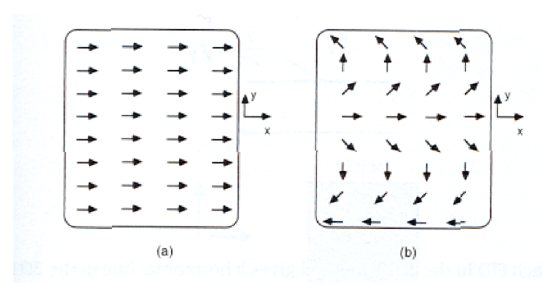
$$e^{-j2\pi k_x x} = \cos(2\pi k_x x) - j\sin(2\pi k_x x)$$



Interpretation



Interpretation



$k_x=0; k_y=0$ $k_x=0; k_y\neq0$

Fig 3.12 from Nishimura

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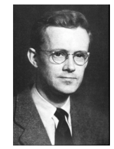
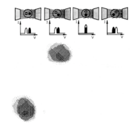
History of MRI

1946: Felix Bloch (Stanford) and Edward Purcell (Harvard) demonstrate nuclear magnetic resonance (NMR)

1973: Paul Lauterbur (SUNY) published first MRI image in Nature.

History of MRI

Late 1970's: First human MRI images

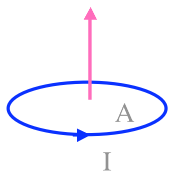
Early 1980's: First commercial MRI systems

1993: functional MRI in humans demonstrated

Spin

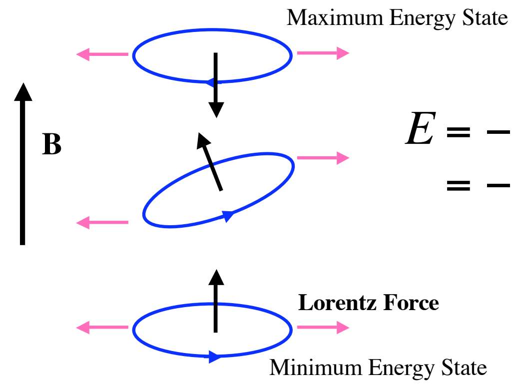
- Intrinsic angular momentum of elementary particles -- electrons, protons, neutrons.
- Spin is quantized. Key concept in Quantum Mechanics.

Classical Magnetic Moment



$$\vec{\mu} = \mathrm{I}\mathrm{A}\hat{\mathrm{n}}$$

Energy in a Magnetic Field



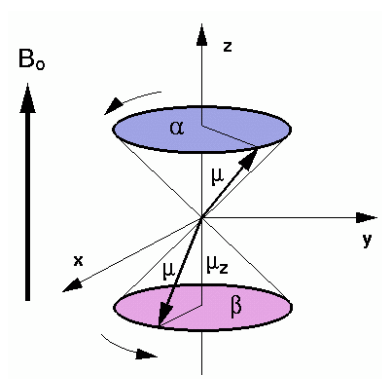
$$E = -\vec{\mu} \bullet \mathbf{B}$$
$$= -\mu_z B$$

Quantization of Magnetic Moment

The key finding of the Stern-Gerlach experiment is that the magnetic moment is quantized. That is, it can only take on discrete values.

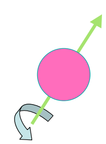
In the experiment, the finding was that the component of magnetization along the direction of the applied field was quantized:

$\mu_z = +\mu_0$ OR $-\mu_0$



http://www.le.ac.uk/biochem/mp84/teaching

Magnetic Moment and Angular Momentum

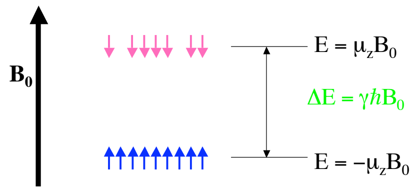


A charged sphere spinning about its axis has angular momentum and a magnetic moment.

This is a classical analogy that is useful for understanding quantum spin, but remember that it is only an analogy!

Relation: $\boldsymbol{\mu} = \gamma\, \mathbf{S}$ where γ is the gyromagnetic ratio and $\mathbf{S}$ is the spin angular momentum.

Boltzmann Distribution



$$\frac{\text{Number Spins Up}}{\text{Number Spins Down}} = \exp(-\Delta E/kT)$$

Ratio = 0.999990 at 1.5T !!!
Corresponds to an excess of about 10 up spins per million

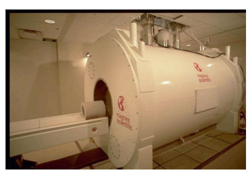
Equilibrium Magnetization

$$\mathbf{M}_0 = N\langle \mu_z \rangle = N\left(\frac{n_{up}(-\mu_z) + n_{down}(\mu_z)}{N}\right)$$
$$= N\mu \frac{e^{\mu_z B/kT} - e^{-\mu_z B/kT}}{e^{\mu_z B/kT} + e^{-\mu_z B/kT}}$$
$$\approx N\mu_z^2 B/(kT)$$
$$= N\gamma^2 \hbar^2 B/(4kT)$$

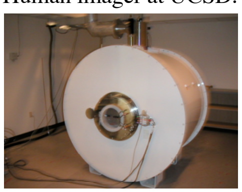
N = number of nuclear spins per unit volume
Magnetization is proportional to applied field.

Bigger is better

3T Human imager at UCSD.



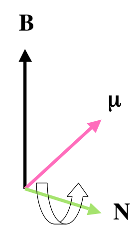
7T Human imager at U. Minn.



7T Rodent Imager at UCSD

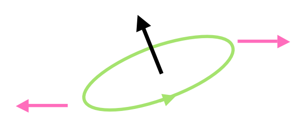
9.4T Human imager at UIC

Torque



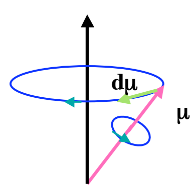
N = μ x B
Torque

For a non-spinning magnetic moment, the torque will try to align the moment with magnetic field (e.g. compass needle)



Precession

$$\frac{d\mu}{dt} = \mu \times \gamma \mathbf{B}$$

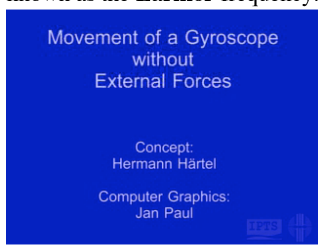


Analogous to motion of a gyroscope

Precesses at an angular frequency of

$$\omega = \gamma \mathbf{B}$$

This is known as the **Larmor** frequency.



http://www.astrophysik.uni-kiel.de/~hhaertel/mpg_e/gyros_free.mpg

Larmor Frequency

$\omega = \gamma \mathbf{B}$ Angular frequency in rad/sec

$f = \gamma \mathbf{B} / (2\pi)$ Frequency in cycles/sec or Hertz, Abbreviated Hz

For a 1.5 T system, the Larmor frequency is 63.86 MHz which is 63.86 million cycles per second. For comparison, KPBS-FM transmits at 89.5 MHz.

Note that the earth's magnetic field is about 50 μT, so that a 1.5T system is about 30,000 times stronger.

Magnetization Vector

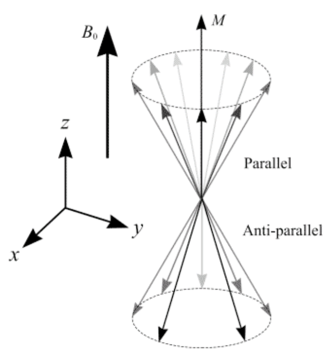
Vector sum of the magnetic moments over a volume.

For a sample at equilibrium in a magnetic field, the transverse components of the moments cancel out, so that there is only a longitudinal component.

Equation of motion is the same form as for individual moments.

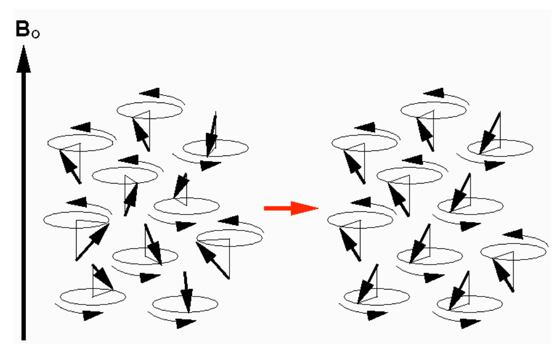
$$\mathbf{M} = \frac{1}{V} \sum_{\substack{\text{protons} \\ \text{in V}}} \mu_i$$

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}$$



http://www.easymeasure.co.uk/principlesmri.aspx

RF Excitation



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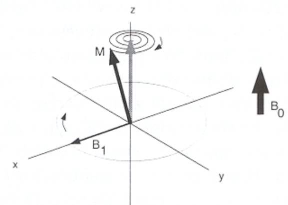
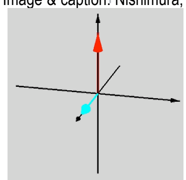


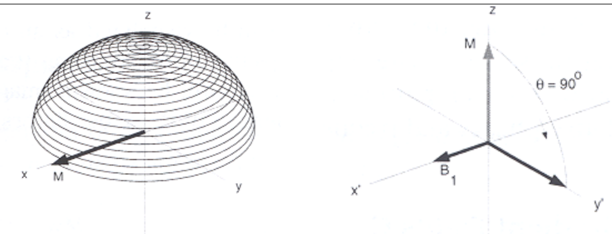
At equilibrium, net magnetizaion is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

Image & caption: Nishimura, Fig. 3.2



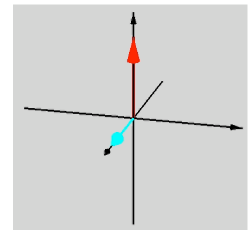
B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (*xy*) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the *z*-axis.
 - lab frame of reference

http://www.eecs.umich.edu/%7Ednoll/BME516/



a) Laboratory frame behavior of **M**   b) Rotating frame behavior of **M**

Images & caption: Nishimura, Fig. 3.3



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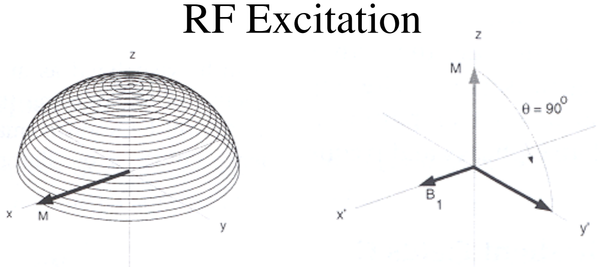
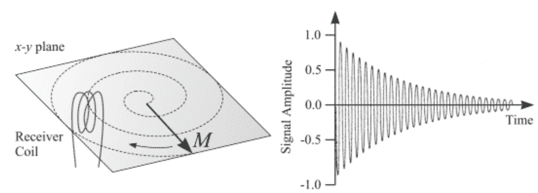


a) Laboratory frame behavior of **M**   b) Rotating frame behavior of **M**

B_1 induces rotation of magnetization towards the transverse plane. Strength and duration of B_1 can be set for a 90 degree rotation, leaving **M** entirely in the *xy* plane. Images & caption: Nishimura, Fig. 3.3

Free Induction Decay (FID)



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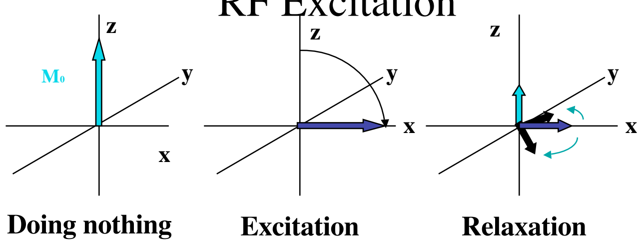
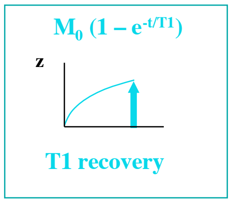
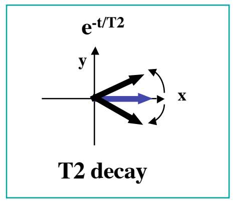






Credit: Larry Frank

Relaxation

An excitation pulse rotates the magnetization vector away from its equilibrium state (purely longitudinal). The resulting vector has both longitudinal $\mathbf{M}_z$ and tranverse $\mathbf{M}_{xy}$ components.

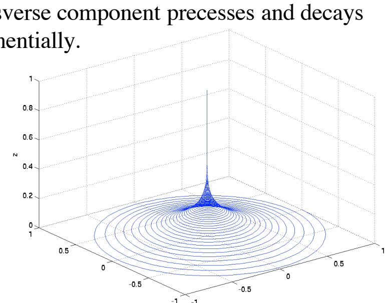
Due to thermal interactions, the magnetization will return to its equilibrium state with characteristic time constants.

T_1 spin-lattice time constant, return to equilibrium of $\mathbf{M}_z$

T_2 spin-spin time constant, return to equilibrium of $\mathbf{M}_{xy}$

Relaxation

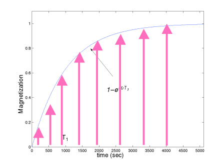
1) Longitudinal component recovers exponentially.
2) Transverse component precesses and decays exponentially.



Fact: Can show that $T_2 < T_1$ in order for $|M(t)| \leq M_0$
Physically, the mechanisms that give rise to T_1 relaxation also contribute to transverse T_2 relaxation.

Longitudinal Relaxation

$$\frac{d\mathbf{M}_z}{dt} = -\frac{M_z - M_0}{T_1}$$

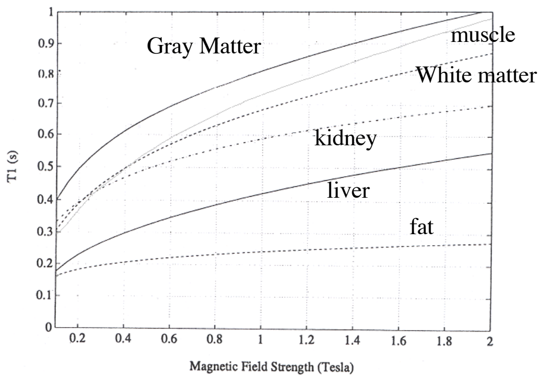


After a 90 degree pulse $M_z(t) = M_0(1 - e^{-t/T_1})$

Due to exchange of energy between nuclei and the lattice (thermal vibrations). Process continues until thermal equilibrium as determined by Boltzmann statistics is obtained.

The energy ΔE required for transitions between down to up spins, increases with field strength, so that T_1 increases with **B**.

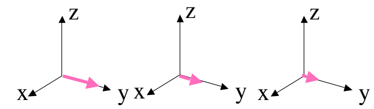
T1 Values



Image, caption: Nishimura, Fig. 4.2

Transverse Relaxation

$$\frac{d\mathbf{M}_{xy}}{dt} = -\frac{M_{xy}}{T_2}$$

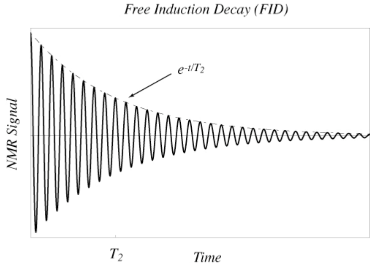


Each spin's local field is affected by the z-component of the field due to other spins. Thus, the Larmor frequency of each spin will be slightly different. This leads to a dephasing of the transverse magnetization, which is characterized by an exponential decay.

T_2 is largely independent of field. T_2 is short for low frequency fluctuations, such as those associated with slowly tumbling macromolecules.

T2 Relaxation

Free Induction Decay (FID)



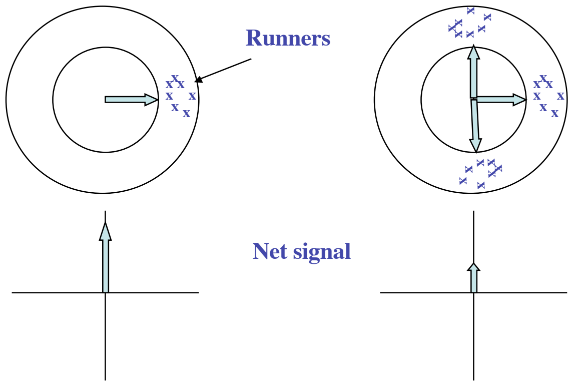
After a 90 degree excitation

$$M_{xy}(t) = M_0 e^{-t/T_2}$$

There is nothing that nuclear spins will not do for you, as long as you treat them as human beings.

Erwin Hahn

T2 Relaxation



Credit: Larry Frank

T2 Values

Tissue	T_2 (ms)
gray matter	100
white matter	92
muscle	47
fat	85
kidney	58
liver	43
CSF	4000

Table: adapted from Nishimura, Table 4.2

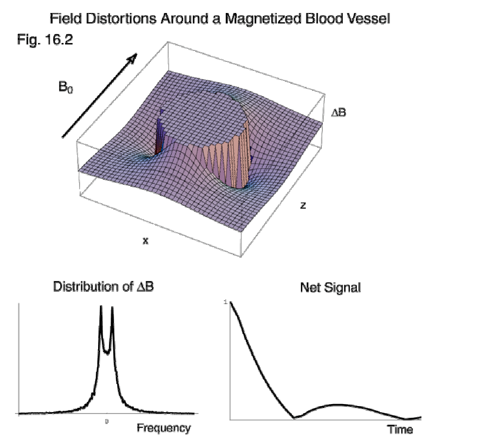
Solids exhibit very short T_2 relaxation times because there are many low frequency interactions between the immobile spins.

On the other hand, liquids show relatively long T_2 values, because the spins are highly mobile and net fields average out.

Static Inhomogeneities

In the ideal situation, the static magnetic field is totally uniform and the reconstructed object is determined solely by the applied gradient fields. In reality, the magnet is not perfect and will not be totally uniform. Part of this can be addressed by additional coils called “shim” coils, and the process of making the field more uniform is called “shimming”. In the old days this was done manually, but modern magnets can do this automatically.

In addition to magnet imperfections, most biological samples are inhomogeneous and this will lead to inhomogeneity in the field. This is because, each tissue has different magnetic properties and will distort the field.



Buxton 2002

T_2^* decay

The overall decay has the form.

$$\exp\left(-t/T_2^*(\vec{r})\right)$$

where

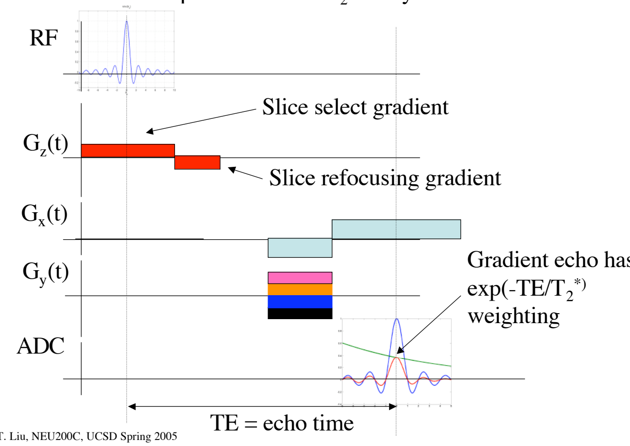
$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

Due to random motions of spins. Not reversible.

Due to static inhomogeneities. Reversible with a spin-echo sequence.

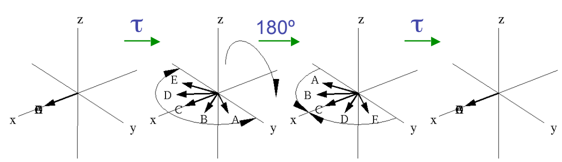
T_2^* decay

Gradient echo sequences exhibit T_2^* decay.



Spin Echo

Discovered by Erwin Hahn in 1950.



The spin-echo can refocus the dephasing of spins due to static inhomogeneities. However, there will still be T_2 dephasing due to random motion of spins.

Image: Larry Frank

Spin Echo Pulse Sequence

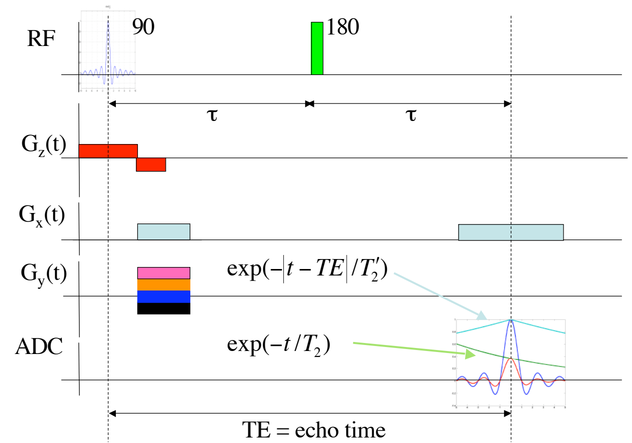
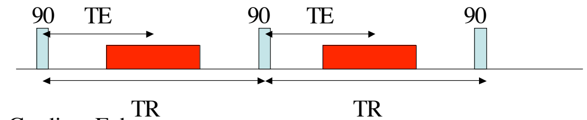


Image Contrast

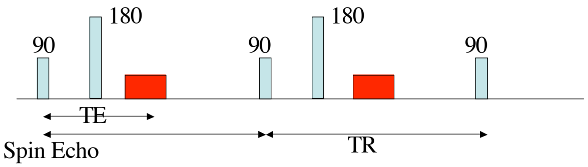
Different tissues exhibit different relaxation rates, T_1, T_2, and T_2^*. In addition different tissues can have different densities of protons. By adjusting the pulse sequence, we can create contrast between the tissues. The most basic way of creating contrast is adjusting the two sequence parameters: TE (echo time) and TR (repetition time).

Saturation Recovery Sequence



Gradient Echo

$$I(x,y) = \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]e^{-TE/T_2^*(x,y)}$$



Spin Echo

$$I(x,y) = \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]e^{-TE/T_2(x,y)}$$

T1-Weighted Scans

Make TE very short compared to either T_2 or T_2^*. The resultant image has both proton and T_1 weighting.

$$I(x,y) \approx \rho(x,y)\left[1 - e^{-TR/T_1(x,y)}\right]$$

T2-Weighted Scans

Make TR very long compared to T_1 and use a spin-echo pulse sequence. The resultant image has both proton and T_2 weighting.

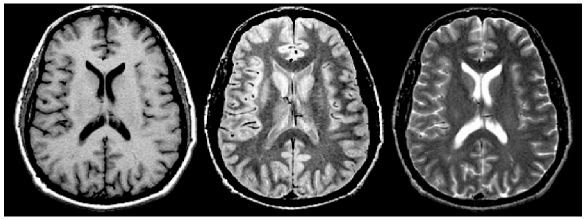
$$I(x,y) \approx \rho(x,y)e^{-TE/T_2}$$

Proton Density Weighted Scans

Make TR very long compared to T_1 and use a very short TE. The resultant image is proton density weighted.

$$I(x,y) \approx \rho(x,y)$$

Example



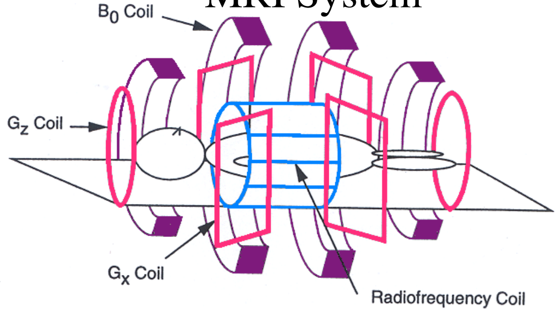
T_1-weighted Density-weighted T_2-weighted

Gradients

Spins precess at the Larmor frequency, which is proportional to the local magnetic field. In a constant magnetic field $B_z = B_0$, all the spins precess at the same frequency (ignoring chemical shift).

Gradient coils are used to add a spatial variation to B_z such that $B_z(x,y,z) = B_0 + \Delta B_z(x,y,z)$. Thus, spins at different physical locations will precess at different frequencies.

MRI System

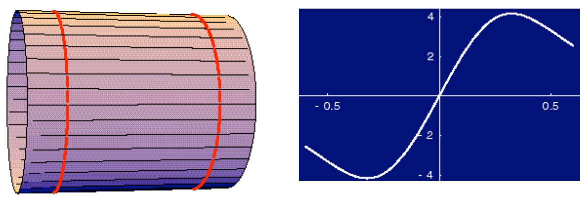


Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field B_o,
gradient fields (two of three shown),
and radiofrequency field B_1.

Image, caption: copyright Nishimura, Fig. 3.15

Z Gradient Coil

B(mT)



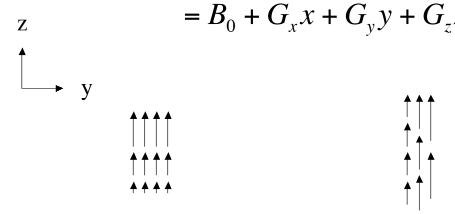
L

Credit: Buxton 2002

Gradient Fields

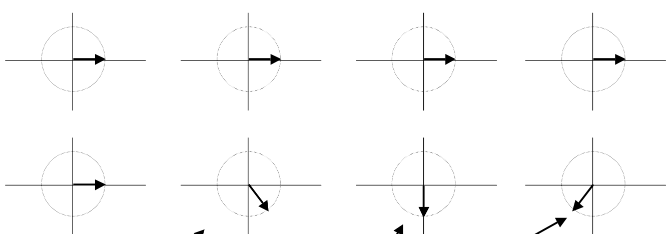
$$B_z(x,y,z) = B_0 + \frac{\partial B_z}{\partial x}x + \frac{\partial B_z}{\partial y}y + \frac{\partial B_z}{\partial z}z$$

$$= B_0 + G_x x + G_y y + G_z z$$



$$G_z = \frac{\partial B_z}{\partial z} > 0 \qquad G_y = \frac{\partial B_z}{\partial y} > 0$$

Precession with Gradient Field

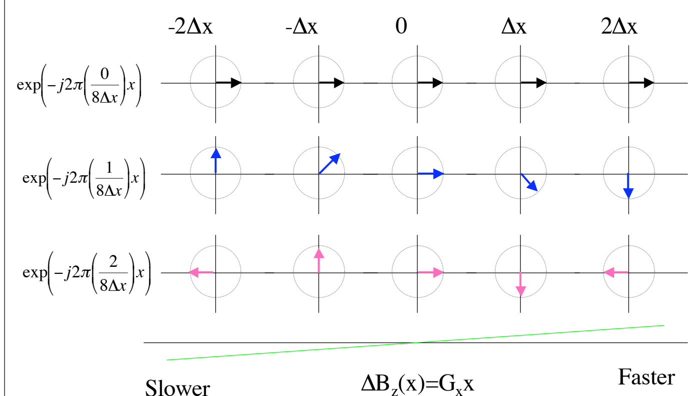


$$\Delta\varphi(\vec{r},t_1) = -\int_0^{t_1} \Delta\omega(\vec{r},\tau)d\tau$$

$$\Delta\varphi(\vec{r},t_2) = -\int_0^{t_2} \Delta\omega(\vec{r},\tau)d\tau$$
$$= -\Delta\omega(\vec{r})t_2$$
if $\Delta\omega$ is non - time varying.

$$\Delta\varphi(\vec{r},t_3) = -\int_0^{t_3} \Delta\omega(\vec{r},\tau)d\tau$$

Interpretation



K-space

At each point in time, the received signal is the Fourier transform of the object

$$s(t) = M\big(k_x(t), k_y(t)\big) = F\big[m(x,y)\big]\Big|_{k_x(t),k_y(t)}$$

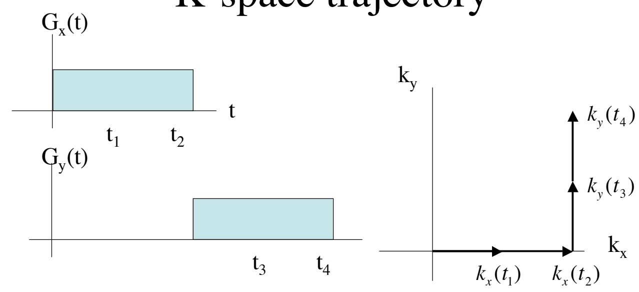
evaluated at the spatial frequencies:

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

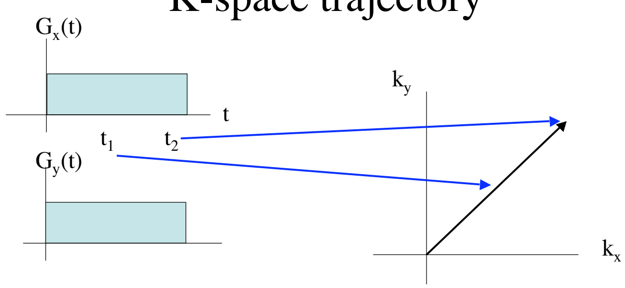
$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

Thus, the gradients control our position in k-space. The design of an MRI pulse sequence requires us to efficiently cover enough of k-space to form our image.

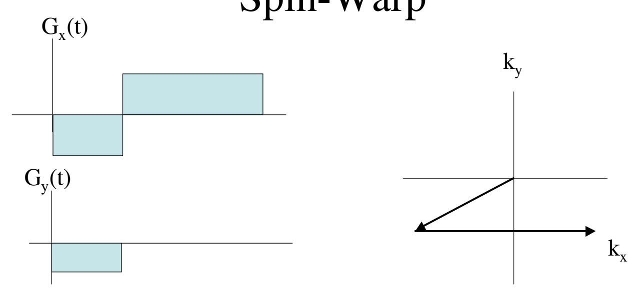
K-space trajectory



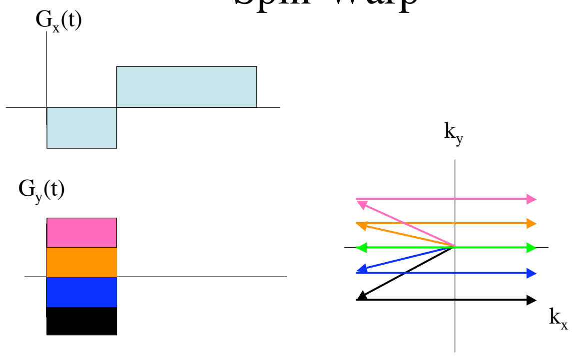
K-space trajectory



Spin-Warp



Spin-Warp



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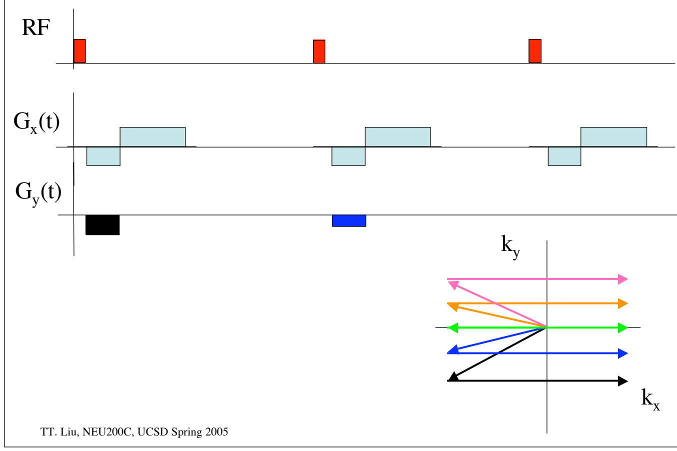
Spin-Warp Pulse Sequence



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Topics

1. Representing Images
2. 2D Fourier Transform
3. MRI Basics
4. **MRI Applications**
5. fMRI

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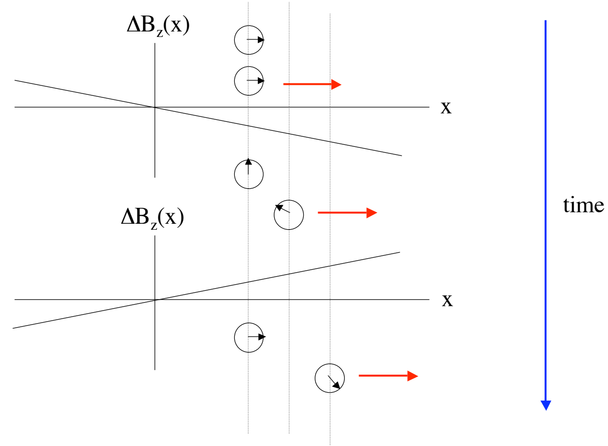
Phase of Moving Spin



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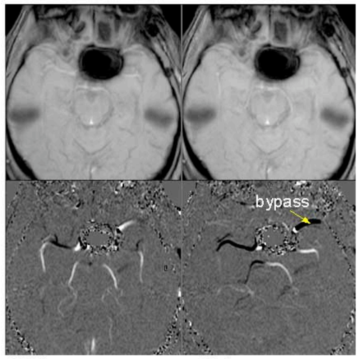
PCA example

$-G_0$

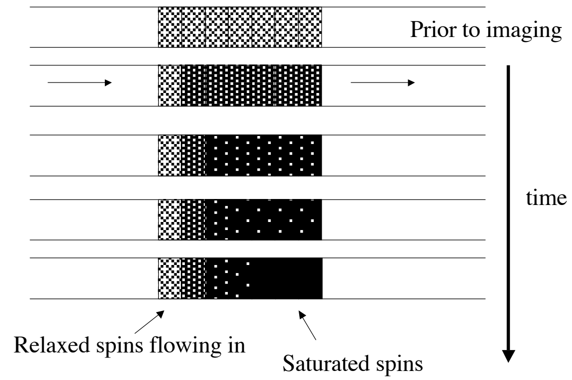


White = Flow direction AP (↓)

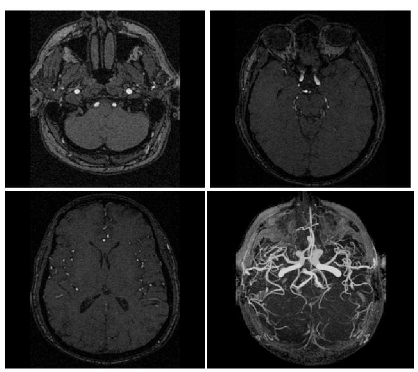
White = Flow direction RL (→)

http://www.medical.philips.com/main/products/mri/assets/images/case_of_week/cotw_51_s5.jpg

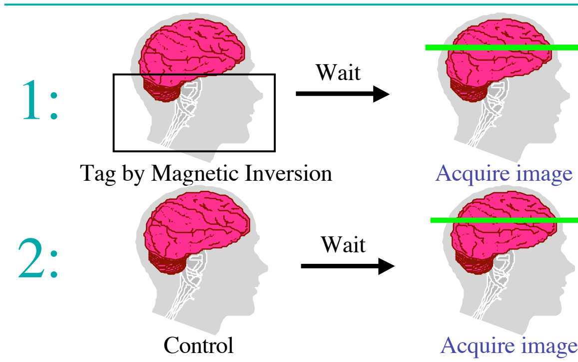
Inflow Effect



Time of Flight Angiography



Arterial Spin Labeling (ASL)



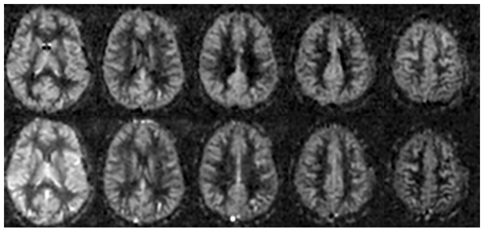
$$\text{Control} - \text{Tag} \propto \text{CBF}$$

Credit: Wen-Ming Luh

Multislice CASL and PICORE

CASL

PICORE
QUIPSS II



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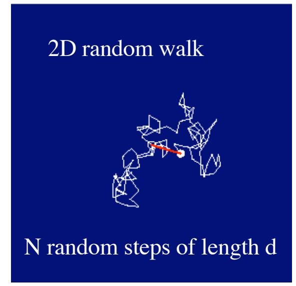
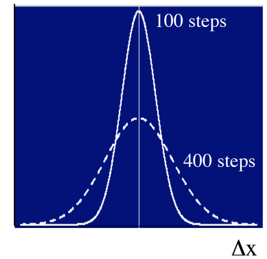
Credit: E. Wong

Diffusion





$<\Delta x^2> = Nd^2 = 2DT$

D = diffusivity

In brain:
$D \cong 0.001\ mm^2/s$
For T=100 msec,
$\Delta x \cong 15\ \mu$

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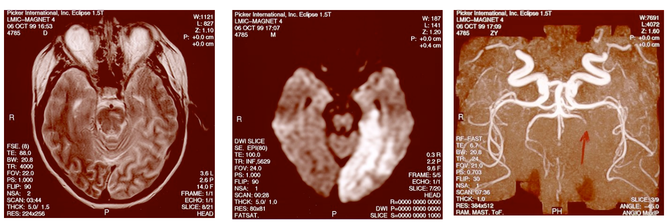
Credit: Larry Frank

Diffusion Weighted Images

T2 weighted | Diffusion Weighted | Angiogram



After a stroke, normal water movement is restricted in the region of damage. Diffusivity decreases, so the signal intensity increases.

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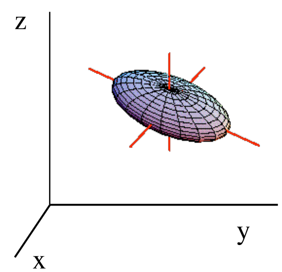
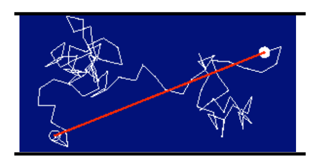
http://lehighmri.com/cases/dwi/patient-b.html

Restricted Diffusion

D depends on direction





Diffusion tensor:
3 values of D
3 angles

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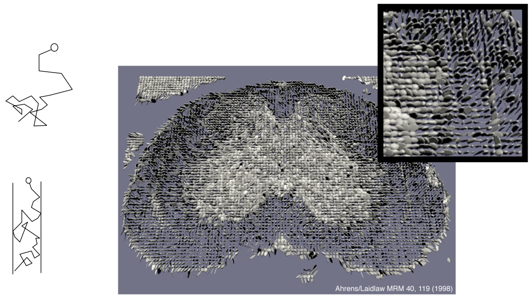
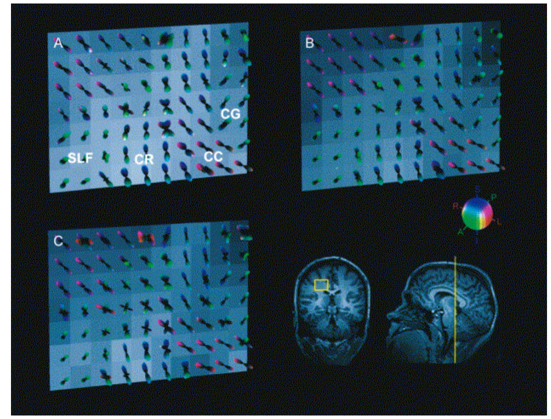
Credit: Larry Frank

Diffusion Imaging Example

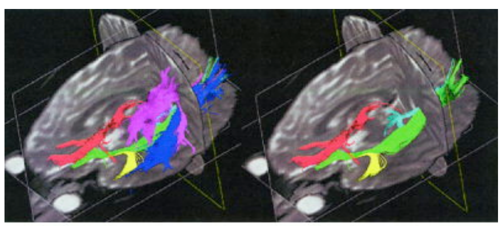


Q-ball imaging



Tuch et al, Neuron 2003

Fiber Tract Mapping



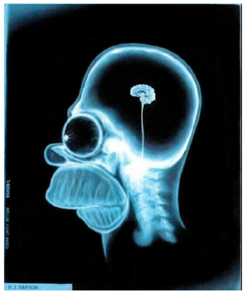
a **b** Mori et al., MRM 2002

Topics

1. Representing Images
2. 2D Fourier Transform
3. MRI Basics
4. MRI Applications
5. **fMRI**

fMRI

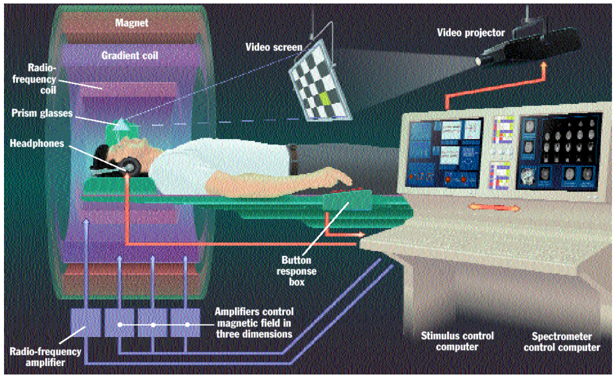
MRI studies brain anatomy.



Functional MRI (fMRI) studies brain function.

http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm

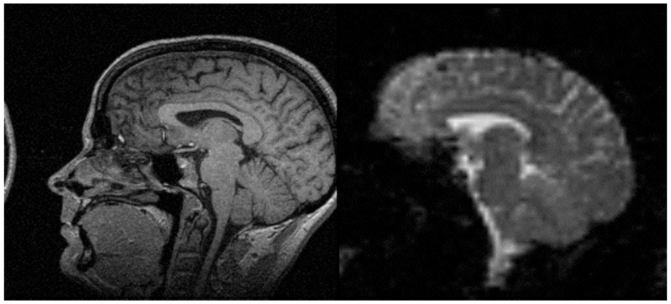
fMRI Setup



http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm

fMRI Acquisition

High spatial resolution | High temporal resolution

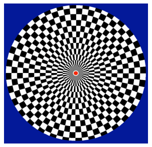


MP-RAGE
Voxel volume: 1 mm^3
Imaging time: 6 min

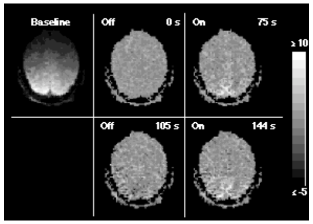
EPI
Voxel volume: 45 mm^3
Imaging time: 60 msec

Buxton 2002

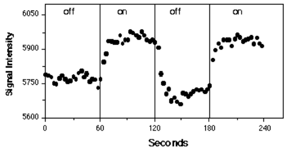
Visual Activation



Flickering Checkerboard
OFF (60 s) - ON (60 s) -OFF (60 s) - ON (60 s) - OFF (60 s)



Brain Activity

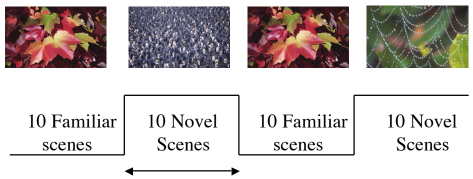


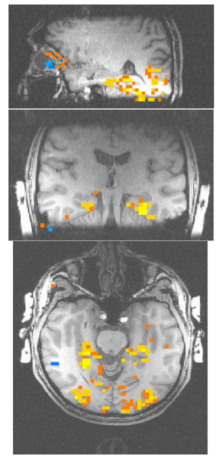
Time ⇨

Source: Kwong et al., 1992

http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm

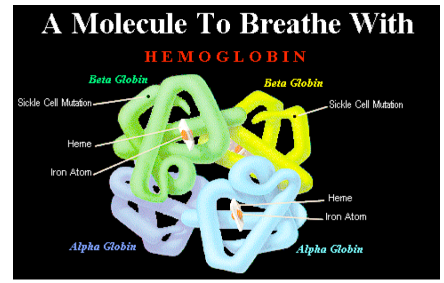
Memory Encoding





The same 4 familiar scenes are used throughout the experiment

Hemoglobin



Oxygen binds to the iron atoms to form oxyhemoglobin HbO_2

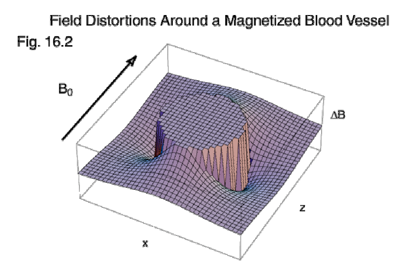
Release of O_2 to tissue results in deoxyhemoglobin $dHBO_2$

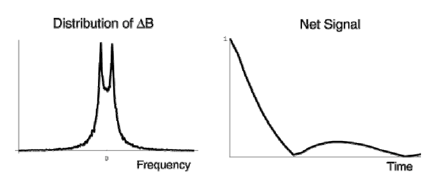
http://www.people.virginia.edu/~rjh9u/hemoglob.html

Effect of $dHBO_2$

$dHBO_2$ is paramagnetic due to the iron atoms. As it becomes oxygenated, it becomes less paramagnetic.

$dHBO_2$ perturbs the local magnetic fields. As blood becomes more deoxygenated, the amount of perturbation increases and there is more dephasing of the spins. Thus as $dHBO_2$ increases we find that T_2^* decreases and the amplitude $\exp(-TE/T_2^*)$ image of a T_2^* weighted image will decrease. Conversely as $dHBO_2$ decreases, T_2^* increases and we expect the signal amplitude to go up.



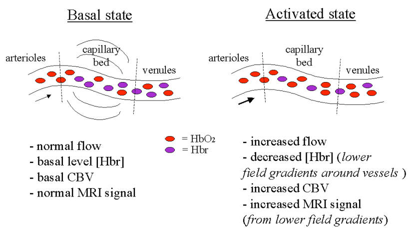


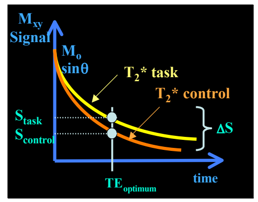
Buxton 2002

BOLD Effect

Blood Oxygen Level Dependent signal

↑neural activity → ↑ blood flow → ↑ oxyhemoglobin → ↑ T2* → ↑ MR signal



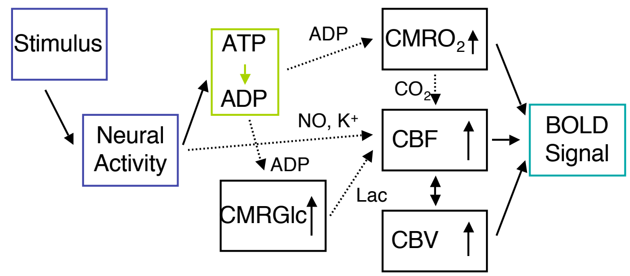


Source: fMRIB Brief Introduction to fMRI

Source: Jorge Jovicich

http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm

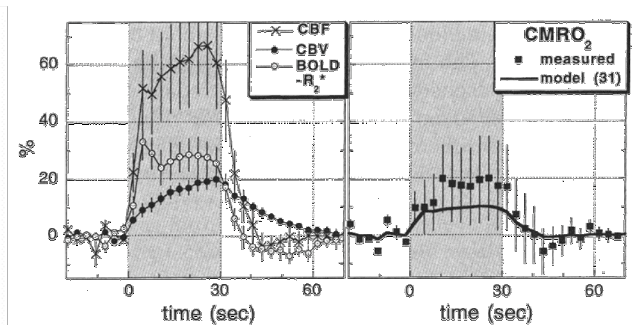
BOLD Dynamics



Credit: Rick Buxton

BOLD Dynamics

Fig. 17.5 Rat Forepaw Stimulation (Mandeville, et al, 1999)



Buxton 2002

BOLD and