Neuroscience 200C

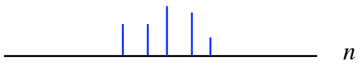
Spring Quarter 2008
Imaging/MRI Lecture

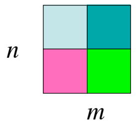
Topics

1. Representing Images
2. Fourier Transform
3. MRI Basics
4. fMRI

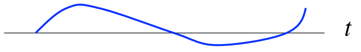
Signals and Images

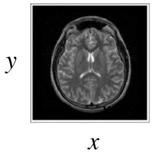
Discrete-time/space signal/image: continuous valued function with a discrete time/space index, denoted as *s[n]* for 1D, *s[m,n]* for 2D , etc.





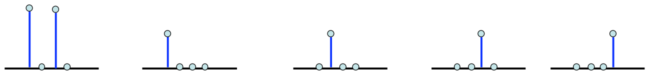
Continuous-time/space signal/image: continuous valued function with a continuous time/space index, denoted as *s(t)* or *s(x)* for 1D, *s(x,y)* for 2D, etc.



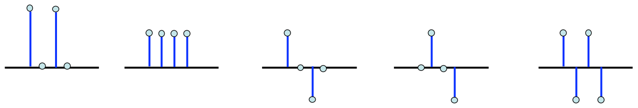


1D Signal Decomposition

$$\{2,0,2,0\} = 2\cdot\{1,0,0,0\} + 0\cdot\{0,1,0,0\} + 2\cdot\{0,0,1,0\} + 0\cdot\{0,0,0,1\}$$



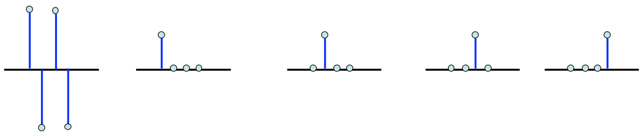
$$\{2,0,2,0\} = a\cdot\{1,1,1,1\} + b\cdot\{1,0,-1,0\} + c\cdot\{0,1,0,-1\} + d\cdot\{1,-1,1,-1\}$$



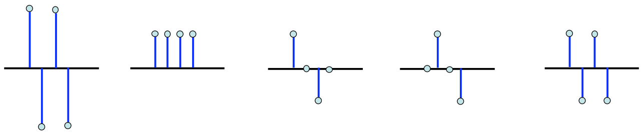
$$\{2,0,2,0\} = 1\cdot\{1,1,1,1\} + 0\cdot\{1,0,-1,0\} + 0\cdot\{0,1,0,-1\} + 1\cdot\{1,-1,1,-1\}$$

1D Signal Decomposition

$$\{2,-2,2,-2\} = 2\cdot\{1,0,0,0\} - 2\cdot\{0,1,0,0\} + 2\cdot\{0,0,1,0\} - 2\cdot\{0,0,0,1\}$$



$$\{2,-2,2,-2\} = 0\cdot\{1,1,1,1\} + 0\cdot\{1,0,-1,0\} + 0\cdot\{0,1,0,-1\} + 2\cdot\{1,-1,1,-1\}$$



Eskimo Words for Snow

tlapa	powder snow
tlacringit	snow that is crusted on the surface
kayi	drifting snow
tlapat	still snow
klin	remembered snow
naklin	forgotten snow
tlamo	snow that falls in large wet flakes
tlatim	snow that falls in small flakes
tlaslo	snow that falls slowly
tlapinti	snow that falls quickly
kripya	snow that has melted and refrozen
tliyel	snow that has been marked by wolves
tliyelin	snow that has been marked by Eskimos
tlalman	snow sold to German tourists
tlalam	snow sold to American tourists
tlanip	snow sold to Japanese tourists
tla-na-na	snow mixed with the sound of old rock and roll from a portable radio
depptla	a small snowball, preserved in Lucite, that had been handled by Johnny Depp

http://www.mendosa.com/snow.html

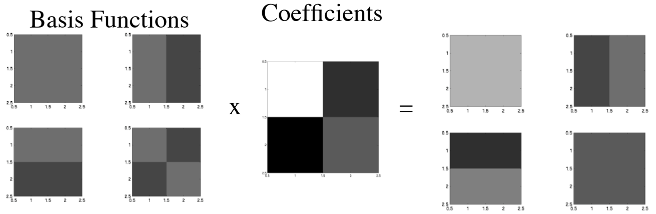
Image Compression

2D Image

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}$$

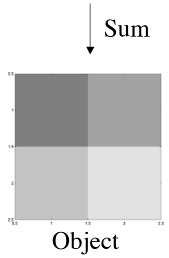
Image Decomposition

$$\begin{array}{|c|c|}\hline a & b \\ \hline c & d \\ \hline\end{array} = a\begin{array}{|c|c|}\hline 1 & 0 \\ \hline 0 & 0 \\ \hline\end{array} + b\begin{array}{|c|c|}\hline 0 & 1 \\ \hline 0 & 0 \\ \hline\end{array} + c\begin{array}{|c|c|}\hline 0 & 0 \\ \hline 1 & 0 \\ \hline\end{array} + d\begin{array}{|c|c|}\hline 0 & 0 \\ \hline 0 & 1 \\ \hline\end{array}$$



$$\begin{array}{|c|c|}\hline 1/2 & 1/2 \\ \hline 1/2 & 1/2 \\ \hline\end{array} \quad \begin{array}{|c|c|}\hline 1/2 & -1/2 \\ \hline 1/2 & -1/2 \\ \hline\end{array}$$

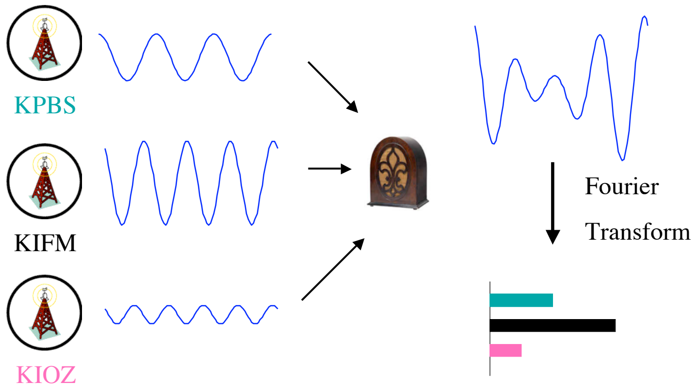
$$\begin{array}{|c|c|}\hline 1/2 & 1/2 \\ \hline -1/2 & -1/2 \\ \hline\end{array} \quad \begin{array}{|c|c|}\hline 1/2 & -1/2 \\ \hline -1/2 & 1/2 \\ \hline\end{array}$$



Topics

1. Representing Images
2. **Fourier Transform**
3. MRI Basics
4. MRI Applications
5. fMRI

1D Fourier Transform



1D Fourier Transform

Fourier Transform

$$G(k_x) = F\left[g(x)\right] = \int_{-\infty}^{\infty} g(x)\exp(-j2\pi k_x x)dx$$

Inverse Fourier Transform

$$g(x) = \int_{-\infty}^{\infty} G(k_x)e^{j2\pi k_x x}dk_x$$

Complex Numbers

$$j = \sqrt{-1}$$

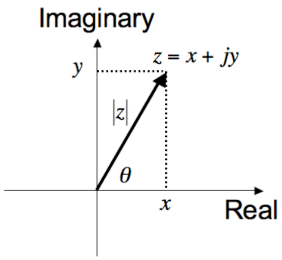
$$j^2 = ?$$

$$(3+2j)(3-2j) = ?$$

$$j^2 = -1$$

$$(3+2j)(3-2j) = 9 - 4j^2 = 13$$

Complex Numbers



$$z = 2 + 1j$$

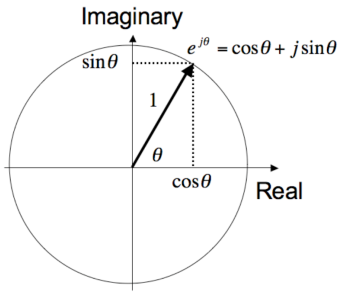
$$|z| = \sqrt{2^2 + 1} = \sqrt{5}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 30 \text{ degrees}$$

Euler's Formula

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$z = x + jy = |z|e^{j\theta}$$



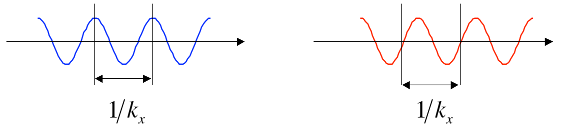
1D Fourier Transform

$$G(k_x)=\int_{-\infty}^{\infty} g(x)\exp(-j2\pi k_x x)dx$$

$$=\int_{-\infty}^{\infty} g(x)\cos(2\pi k_x x)dx - j\int_{-\infty}^{\infty} g(x)\sin(2\pi k_x x)dx$$

The part of $g(x)$ that "looks" like $\cos(2\pi k_x x)$

The part of $g(x)$ that "looks" like $\sin(2\pi k_x x)$



2D Fourier Transform

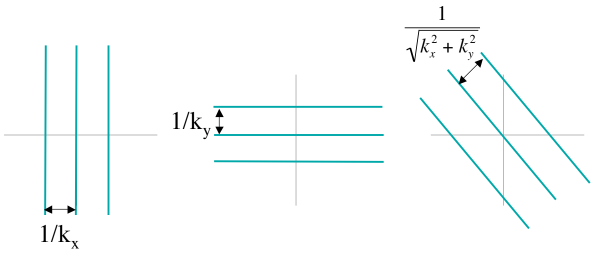
$$G(k_x,k_y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi\left(k_x x+k_y y\right)}dxdy$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\cos\left(2\pi\left(k_x x+k_y y\right)\right)\,dxdy$$

$$-j\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\sin\left(2\pi\left(k_x x+k_y y\right)\right)dxdy$$

2D Plane Waves

$$e^{j2\pi(k_x x+k_y y)}=\cos\left(2\pi(k_x x+k_y y)\right)+j\sin\left(2\pi(k_x x+k_y y)\right)$$



$\cos(2\pi k_x x)$ $\cos(2\pi k_y y)$ $\cos(2\pi k_x x + 2\pi k_y y)$

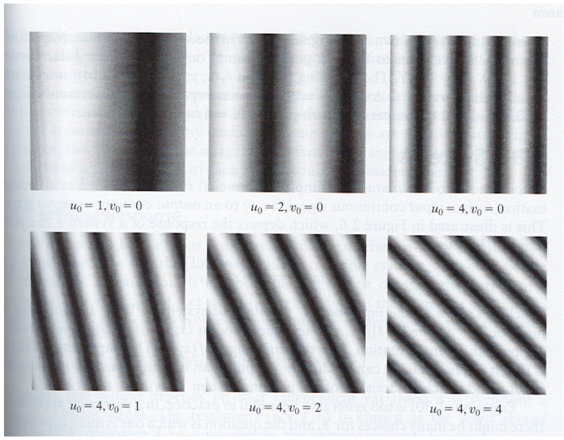
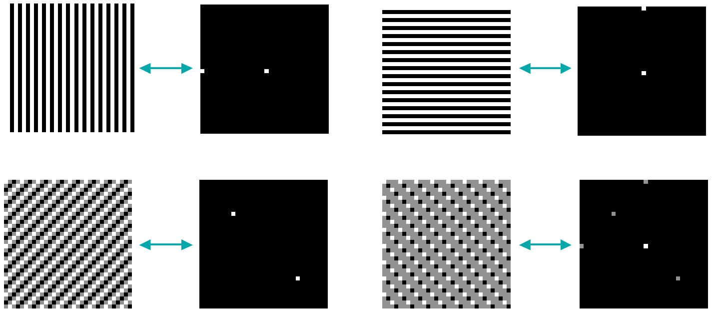


Figure 2.5 from Prince and Link

2D Fourier Transform



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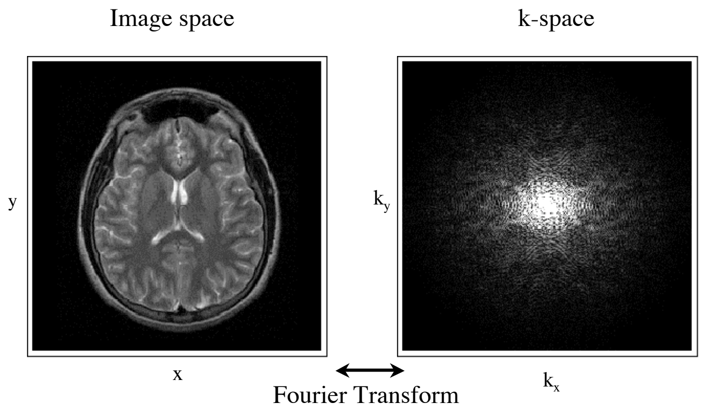
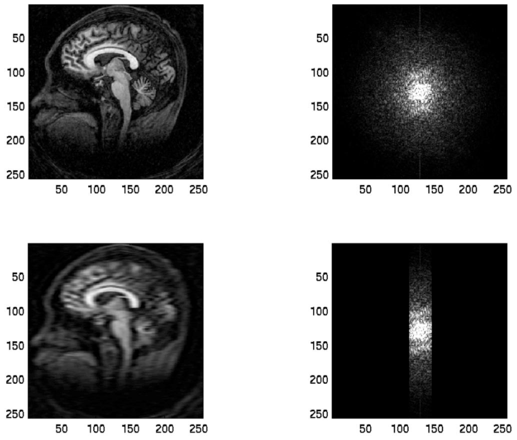
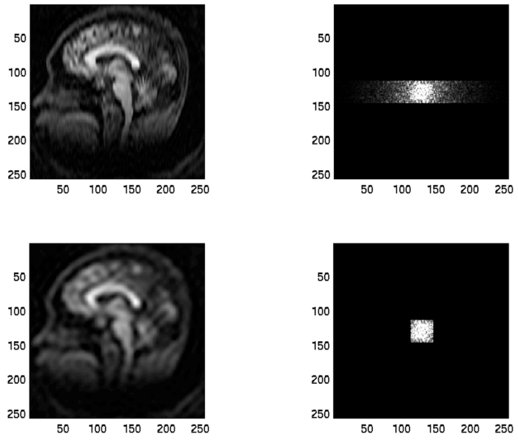
k-space



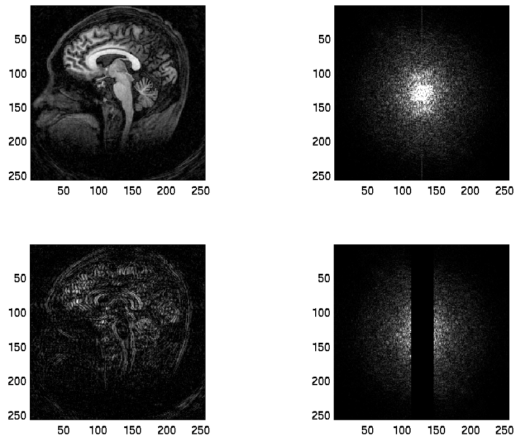
Examples



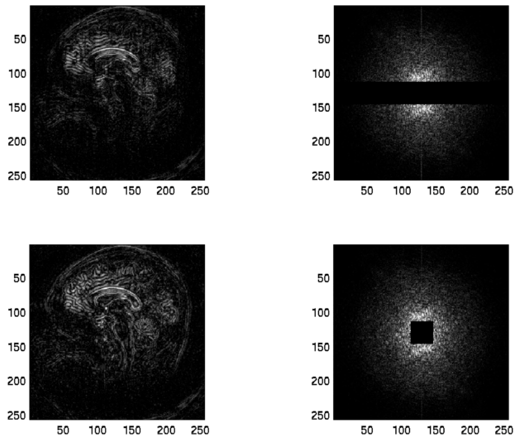
Examples



Examples

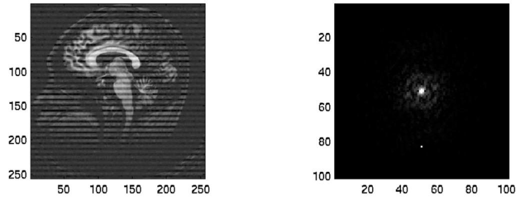


Examples

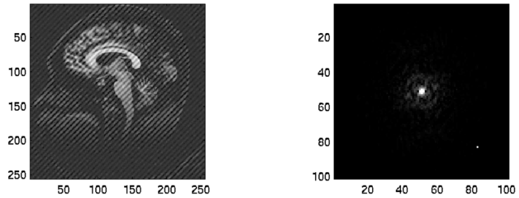


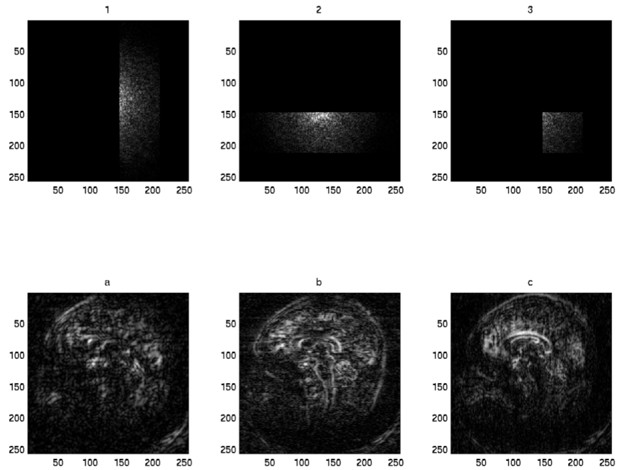
Examples

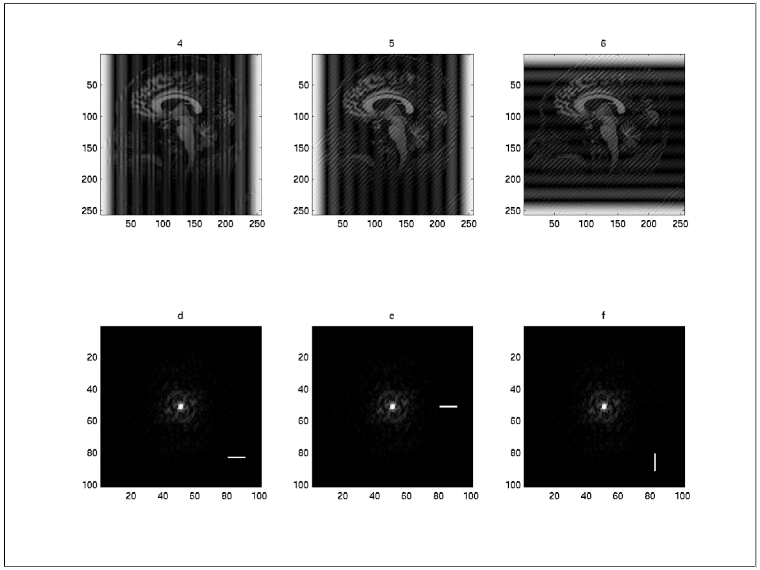
(c) spike off-center in ky

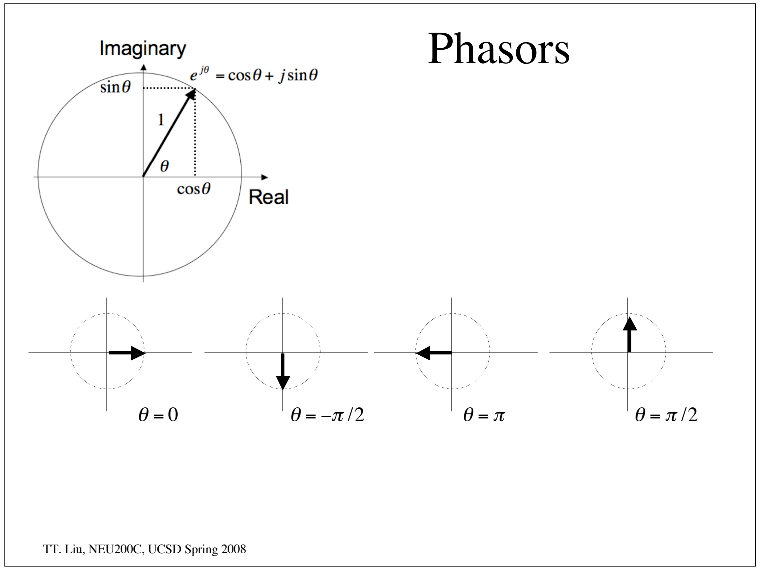


(d) spike off-center in kx





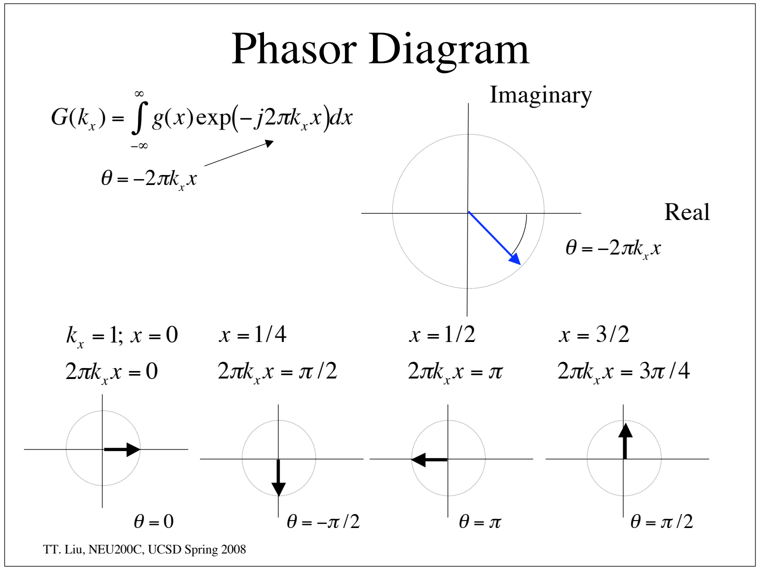




Phasors

$\theta = 0$ $\theta = -\pi/2$ $\theta = \pi$ $\theta = \pi/2$

TT. Liu, NEU200C, UCSD Spring 2008



Phasor Diagram

$$G(k_x) = \int_{-\infty}^{\infty} g(x)\exp(-j2\pi k_x x)dx$$

$\theta = -2\pi k_x x$

$k_x = 1;\ x = 0$	$x = 1/4$	$x = 1/2$	$x = 3/2$
$2\pi k_x x = 0$	$2\pi k_x x = \pi/2$	$2\pi k_x x = \pi$	$2\pi k_x x = 3\pi/4$
$\theta = 0$	$\theta = -\pi/2$	$\theta = \pi$	$\theta = \pi/2$

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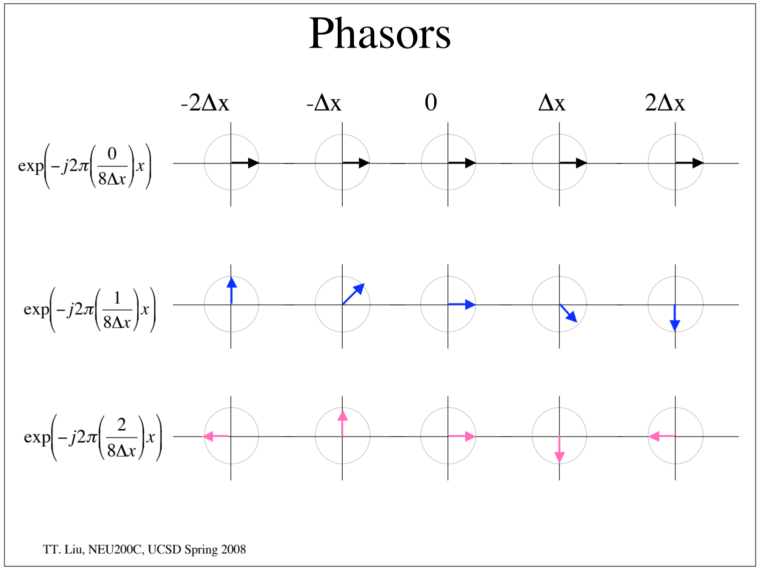

Phasors

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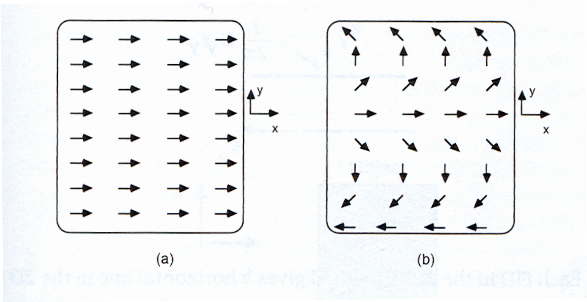
Interpretation



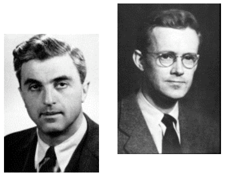
$k_x=0;\ k_y=0$ $k_x=0;\ k_y\neq 0$

Fig 3.12 from Nishimura

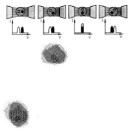
Topics

1. Representing Images
2. 2D Fourier Transform
3. **MRI Basics**
4. fMRI

History of MRI



1946: Felix Bloch (Stanford) and Edward Purcell (Harvard) demonstrate nuclear magnetic resonance (NMR)



1973: Paul Lauterbur (SUNY) published first MRI image in Nature.

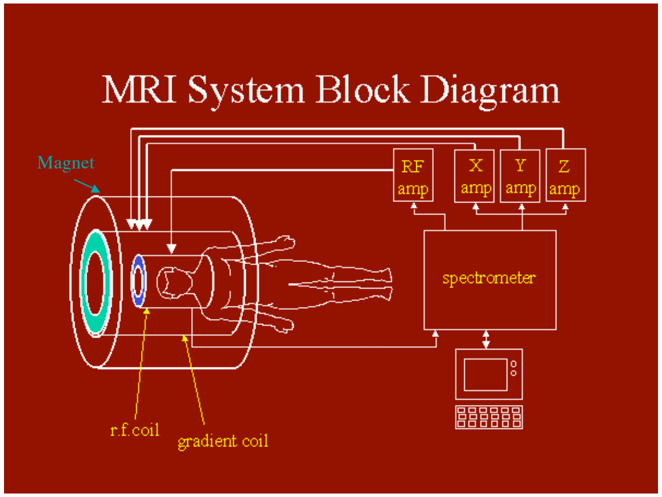
History of MRI

Late 1970's: First human MRI images

Early 1980's: First commercial MRI systems

1993: functional MRI in humans demonstrated

Clinical MRI System



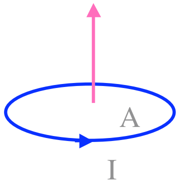
MRI System Block Diagram

Image from http://www.fmrib.ox.ac.uk/~stuart/lectures/lecture1/

Spin

- Intrinsic angular momentum of elementary particles -- electrons, protons, neutrons.
- Spin is quantized. Key concept in Quantum Mechanics.

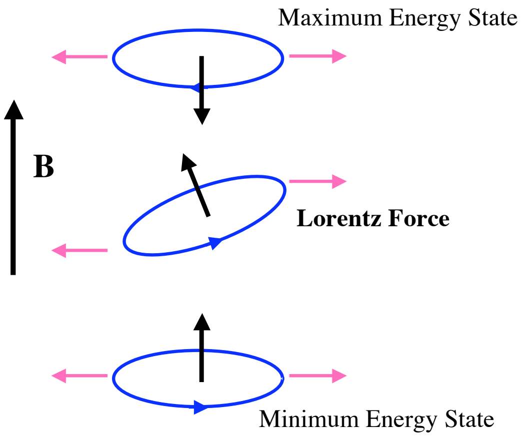
Classical Magnetic Moment

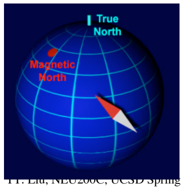


$$\vec{\mu} = IA\hat{n}$$

Energy in a Magnetic Field

$$E = -\vec{\mu} \bullet \mathbf{B}$$
$$= -\mu_z B$$



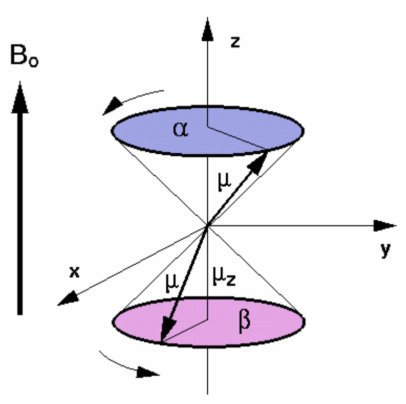


Quantization of Magnetic Moment

The key finding of the Stern-Gerlach experiment is that the magnetic moment is quantized. That is, it can only take on discrete values.

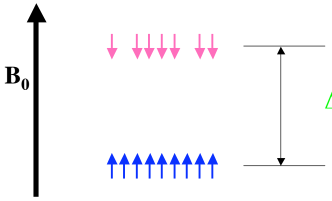
In the experiment, the finding was that the component of magnetization along the direction of the applied field was quantized:

$$\mu_z = +\mu_0 \text{ OR } -\mu_0$$



http://www.le.ac.uk/biochem/mp84/teaching

Boltzmann Distribution



$E = \mu_z B_0$

$\Delta E = \gamma\hbar B_0$

$E = -\mu_z B_0$

$$\frac{\text{Number Spins Up}}{\text{Number Spins Down}} = \exp(-\Delta E/kT)$$

Ratio = 0.999990 at 1.5T !!!

Corresponds to an excess of about 10 up spins per million

Equilibrium Magnetization

$$\begin{aligned}
\mathbf{M}_0 &= N\langle \mu_z \rangle = N\left(\frac{n_{up}(-\mu_z) + n_{down}(\mu_z)}{N}\right) \\
&= N\mu \frac{e^{\mu_z B/kT} - e^{-\mu_z B/kT}}{e^{\mu_z B/kT} + e^{-\mu_z B/kT}} \\
&\approx N\mu_z^2 B/(kT) \\
&= N\gamma^2\hbar^2 B/(4kT)
\end{aligned}$$

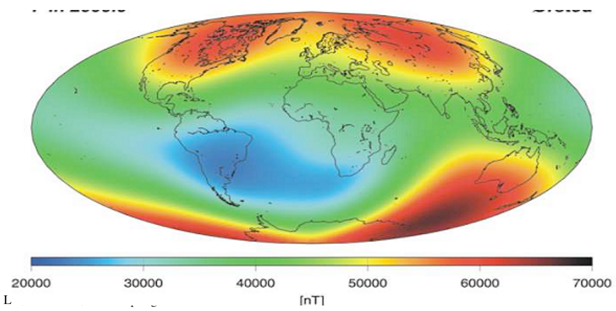
N = number of nuclear spins per unit volume
Magnetization is proportional to applied field.

Magnetic Field Units

1 Tesla = 10,000 Gauss

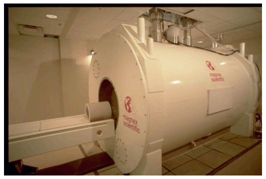
Earth's field is about 0.5 Gauss

0.5 Gauss = 0.5×10^{-4} T = 50 μT

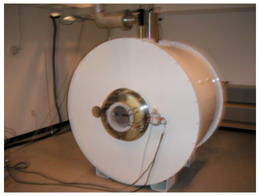


Bigger is better

3T Human imager at UCSD.



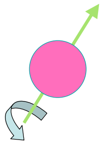
7T Human imager at U. Minn.



7T Rodent Imager at UCSD

9.4T Human imager at UIC

Magnetic Moment and Angular Momentum



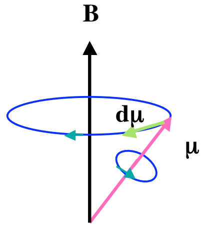
A charged sphere spinning about its axis has angular momentum and a magnetic moment.

This is a classical analogy that is useful for understanding quantum spin, but remember that it is only an analogy!

Relation: $\boldsymbol{\mu} = \gamma \mathbf{S}$ where γ is the gyromagnetic ratio and $\mathbf{S}$ is the spin angular momentum.

Precession

$$\frac{d\boldsymbol{\mu}}{dt} = \boldsymbol{\mu} \times \gamma \mathbf{B}$$



Analogous to motion of a gyroscope

Precesses at an angular frequency of

$\omega = \gamma B$

This is known as the **Larmor** frequency.

Larmor Frequency

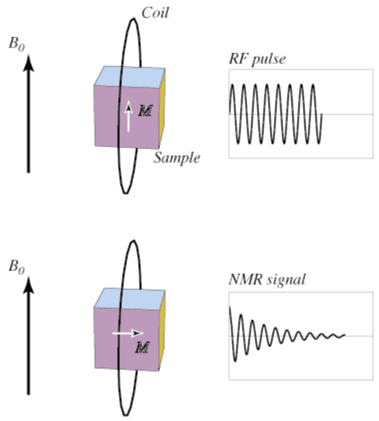
$\omega = \gamma B$ Angular frequency in rad/sec

$f = \gamma B / (2 \pi)$ Frequency in cycles/sec or Hertz, Abbreviated Hz

For a 1.5 T system, the Larmor frequency is 63.86 MHz which is 63.86 million cycles per second. For comparison, KPBS-FM transmits at 89.5 MHz.

Note that the earth's magnetic field is about 50 μT, so that a 1.5T system is about 30,000 times stronger.

RF Excitation



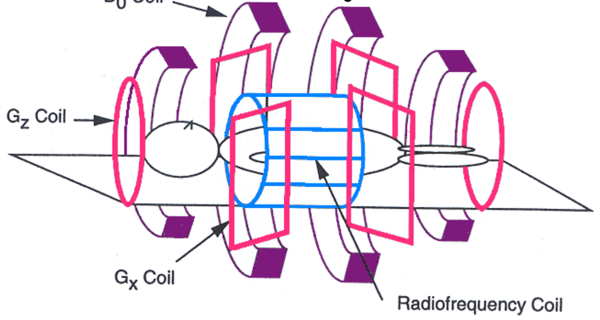
From Buxton 2002

Gradients

Spins precess at the Larmor frequency, which is proportional to the local magnetic field. In a constant magnetic field $B_z = B_0$, all the spins precess at the same frequency (ignoring chemical shift).

Gradient coils are used to add a spatial variation to B_z such that $B_z(x,y,z) = B_0 + \Delta B_z(x,y,z)$. Thus, spins at different physical locations will precess at different frequencies.

MRI System

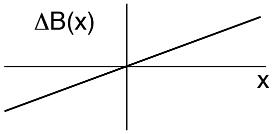


Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field B_o,
gradient fields (two of three shown),
and radiofrequency field B_1.

Image, caption: copyright Nishimura, Fig. 3.15

Imaging: localizing the NMR signal

The local precession frequency can be changed in a position-dependent way by applying linear field gradients



Resonant Frequency:

$\nu(x) = \gamma B_0 + \gamma \Delta B(x)$

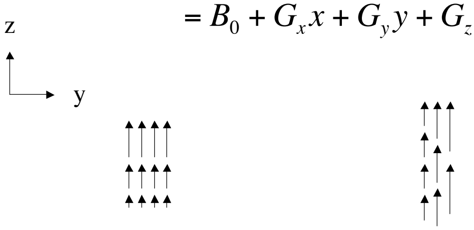
RF and Gradient Coils

Credit: R. Buxton

Gradient Fields

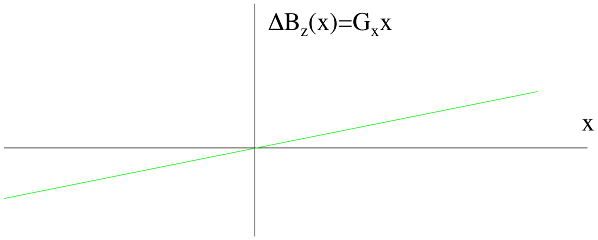
$$B_z(x,y,z) = B_0 + \frac{\partial B_z}{\partial x}x + \frac{\partial B_z}{\partial y}y + \frac{\partial B_z}{\partial z}z$$

$$= B_0 + G_x x + G_y y + G_z z$$



$$G_z = \frac{\partial B_z}{\partial z} > 0 \qquad G_y = \frac{\partial B_z}{\partial y} > 0$$

Interpretation



Spins Precess at $\gamma B_0 - \gamma G_x x$ (slower)

Spins Precess at γB_0

Spins Precess at $\gamma B_0 + \gamma G_x x$ (faster)

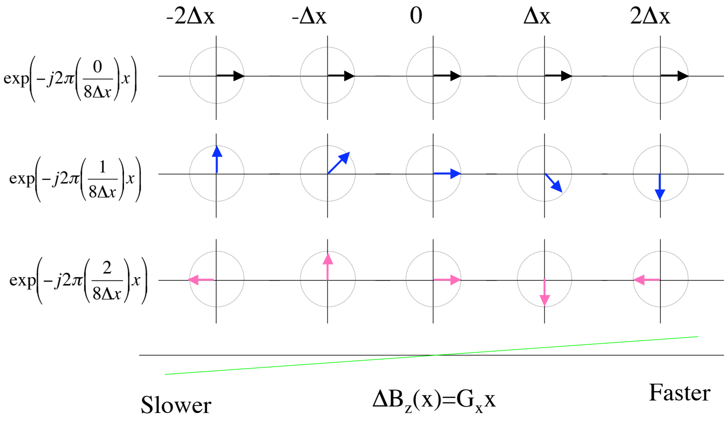
Rotating Frame of Reference

Reference everything to the magnetic field at isocenter.

Spins

There is nothing that nuclear spins will not do for you, as long as you treat them as human beings.
Erwin Hahn

Interpretation

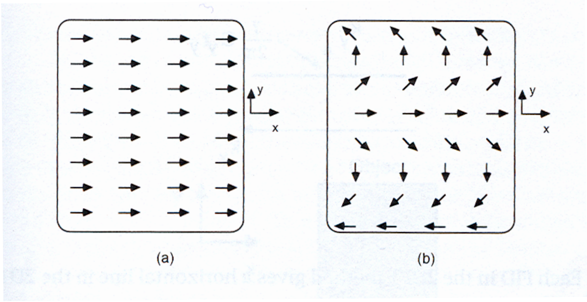


$\exp\left(-j2\pi\left(\frac{0}{8\Delta x}\right)x\right)$

$\exp\left(-j2\pi\left(\frac{1}{8\Delta x}\right)x\right)$

$\exp\left(-j2\pi\left(\frac{2}{8\Delta x}\right)x\right)$

$\Delta B_z(x)=G_x x$

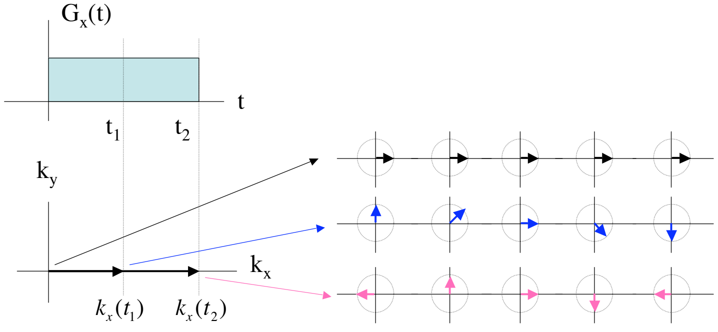


$k_x=0; k_y=0$

$k_x=0; k_y\neq 0$

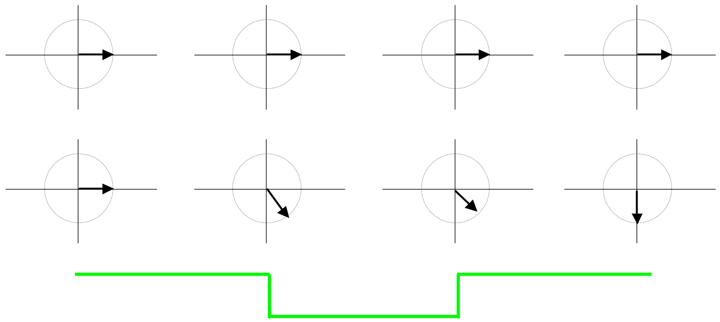
Fig 3.12 from Nishimura

K-space trajectory

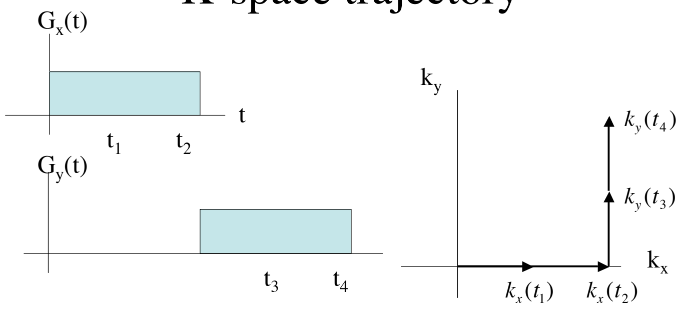


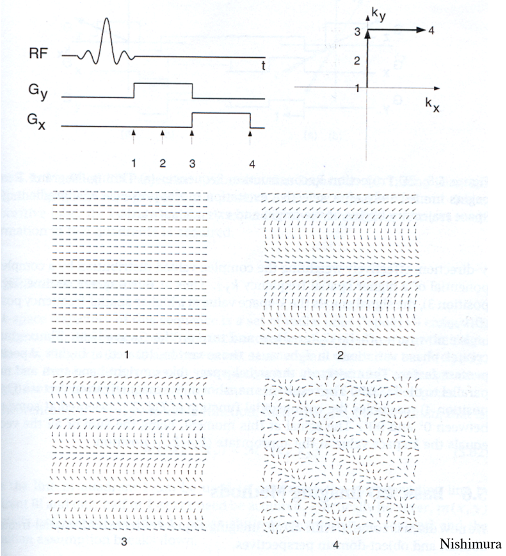
$$k_x(t)=\frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

Phase with time-varying gradient



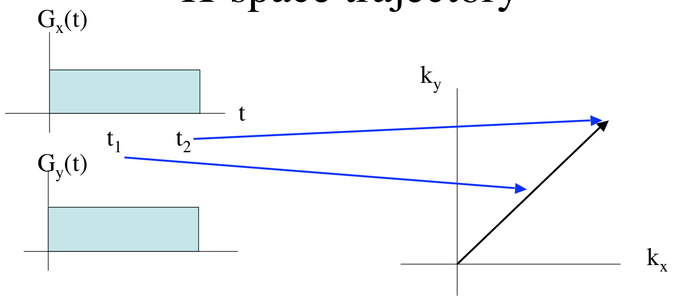
K-space trajectory

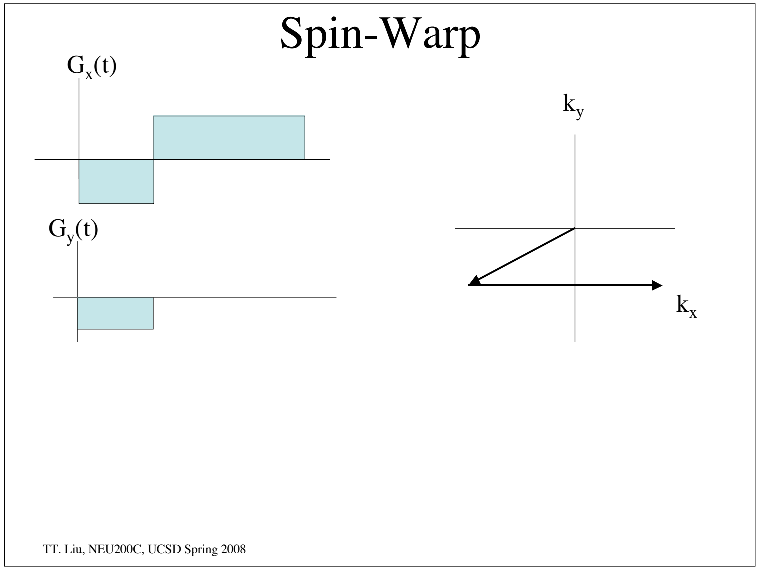




Nishimura 1996

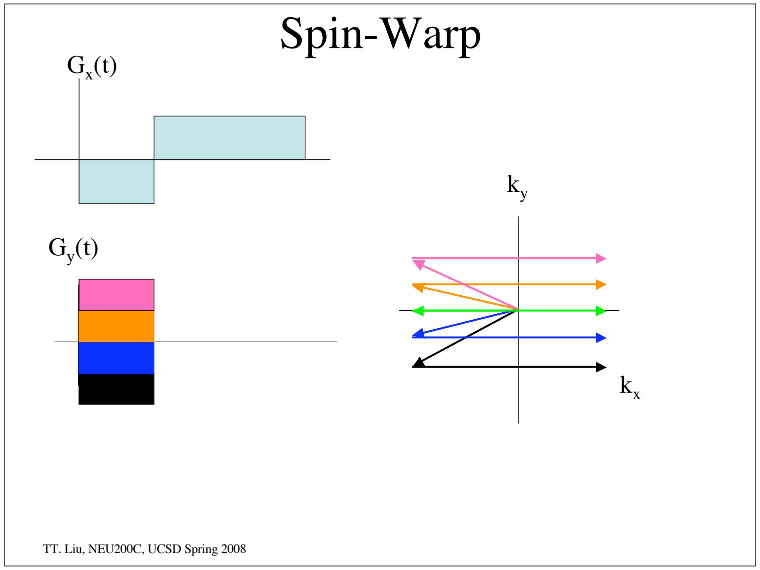
K-space trajectory





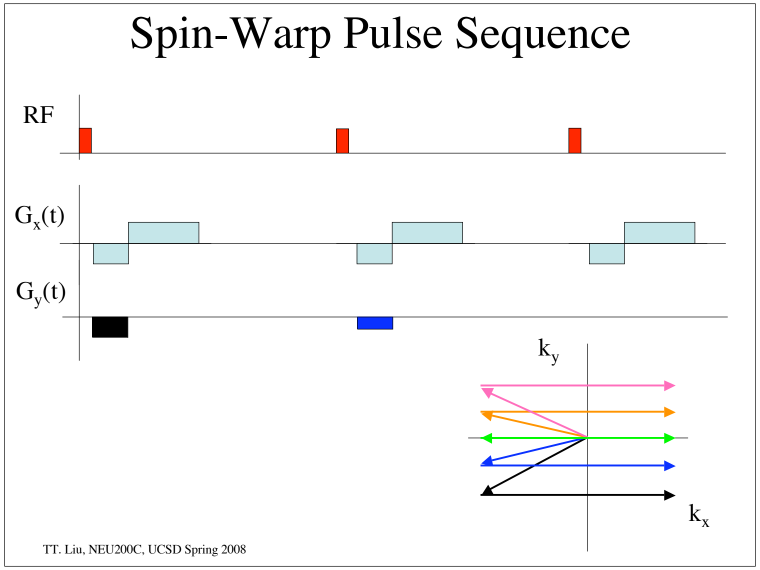
Spin-Warp

TT. Liu, NEU200C, UCSD Spring 2008



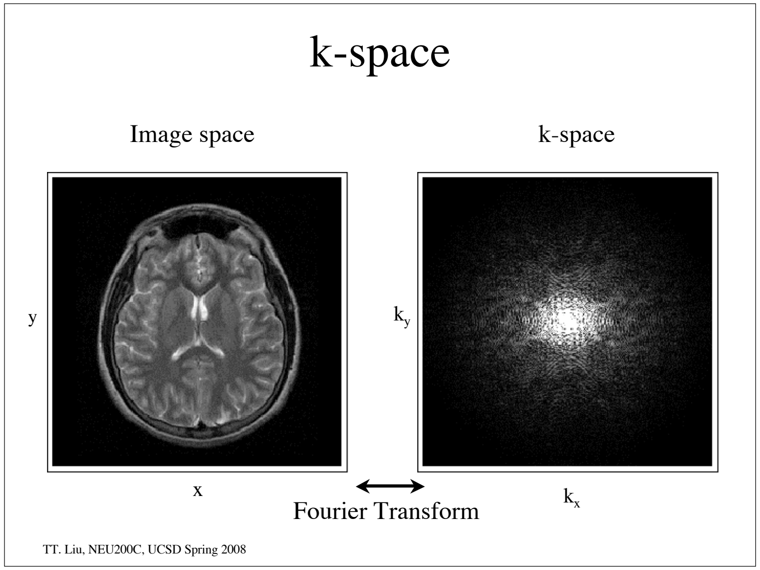
Spin-Warp

TT. Liu, NEU200C, UCSD Spring 2008



Spin-Warp Pulse Sequence

TT. Liu, NEU200C, UCSD Spring 2008



k-space

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Relaxation

An excitation pulse rotates the magnetization vector away from its equilibrium state (purely longitudinal). The resulting vector has both longitudinal $\mathbf{M}_z$ and tranverse $\mathbf{M}_{xy}$ components.

Due to thermal interactions, the magnetization will return to its equilibrium state with characteristic time constants.

T_1 spin-lattice time constant, return to equilibrium of $\mathbf{M}_z$

T_2 spin-spin time constant, return to equilibrium of $\mathbf{M}_{xy}$

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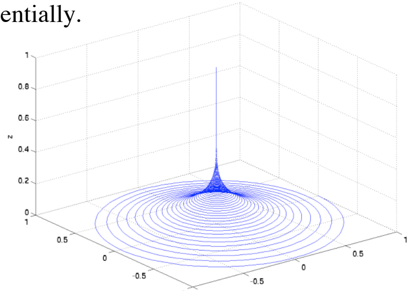
Relaxation

1) Longitudinal component recovers exponentially.
2) Transverse component precesses and decays exponentially.



Fact: Can show that $T_2 < T_1$ in order for $|M(t)| \leq M_0$
Physically, the mechanisms that give rise to T_1 relaxation also contribute to transverse T_2 relaxation.

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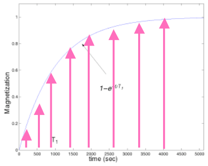
Longitudinal Relaxation

$$\frac{d\mathbf{M}_z}{dt} = -\frac{M_z - M_0}{T_1}$$



After a 90 degree pulse $\quad M_z(t) = M_0(1 - e^{-t/T_1})$

Due to exchange of energy between nuclei and the lattice (thermal vibrations). Process continues until thermal equilibrium as determined by Boltzmann statistics is obtained.

The energy ΔE required for transitions between down to up spins, increases with field strength, so that T_1 increases with **B**.

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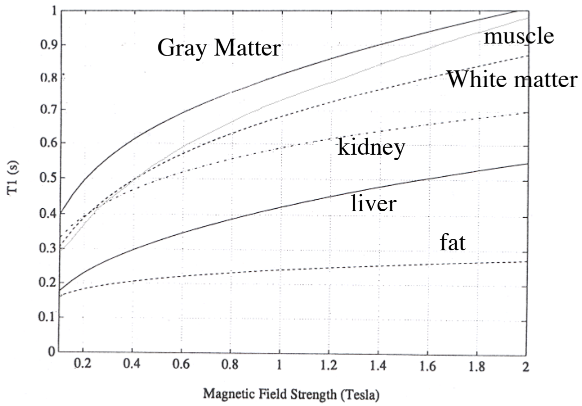
T1 Values

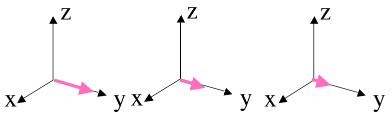


Image, caption: Nishimura, Fig. 4.2

TT. Liu, NEU200C, UCSD Spring 2008

Transverse Relaxation

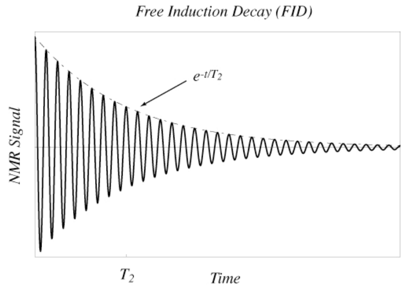
$$\frac{d\mathbf{M}_{xy}}{dt} = -\frac{M_{xy}}{T_2}$$



Each spin's local field is affected by the z-component of the field due to other spins. Thus, the Larmor frequency of each spin will be slightly different. This leads to a dephasing of the transverse magnetization, which is characterized by an exponential decay.

T_2 is largely independent of field. T_2 is short for low frequency fluctuations, such as those associated with slowly tumbling macromolecules.

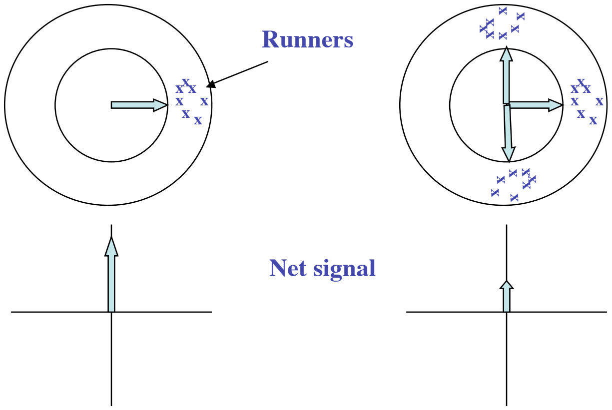
T2 Relaxation



After a 90 degree excitation

$$M_{xy}(t) = M_0 e^{-t/T_2}$$

T2 Relaxation



Credit: Larry Frank

T2 Values

Tissue	T_2 (ms)
gray matter	100
white matter	92
muscle	47
fat	85
kidney	58
liver	43
CSF	4000

Table: adapted from Nishimura, Table 4.2

Solids exhibit very short T_2 relaxation times because there are many low frequency interactions between the immobile spins.

On the other hand, liquids show relatively long T_2 values, because the spins are highly mobile and net fields average out.

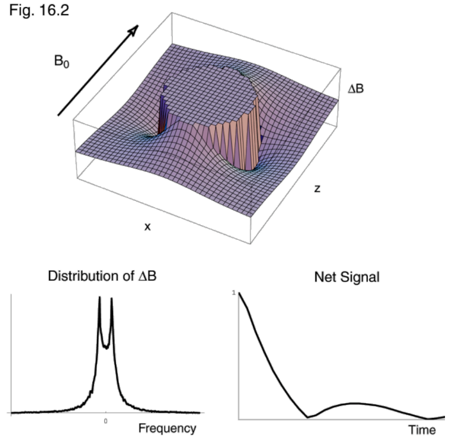
Static Inhomogeneities

In the ideal situation, the static magnetic field is totally uniform and the reconstructed object is determined solely by the applied gradient fields. In reality, the magnet is not perfect and will not be totally uniform. Part of this can be addressed by additional coils called "shim" coils, and the process of making the field more uniform is called "shimming". In the old days this was done manually, but modern magnets can do this automatically.

In addition to magnet imperfections, most biological samples are inhomogeneous and this will lead to inhomogeneity in the field. This is because, each tissue has different magnetic properties and will distort the field.

Field Distortions Around a Magnetized Blood Vessel

Fig. 16.2



Buxton 2002

T_2^* decay

The overall decay has the form.

$$\exp\left(-t/T_2^*(\vec{r})\right)$$

where

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

Due to random motions of spins. Not reversible.

Due to static inhomogeneities. Reversible with a spin-echo sequence.

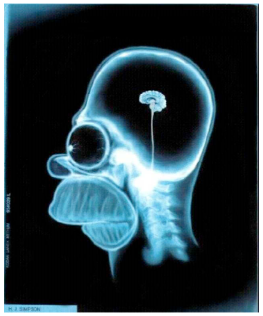
Topics

1. Representing Images
2. 2D Fourier Transform
3. MRI Basics
4. fMRI

fMRI

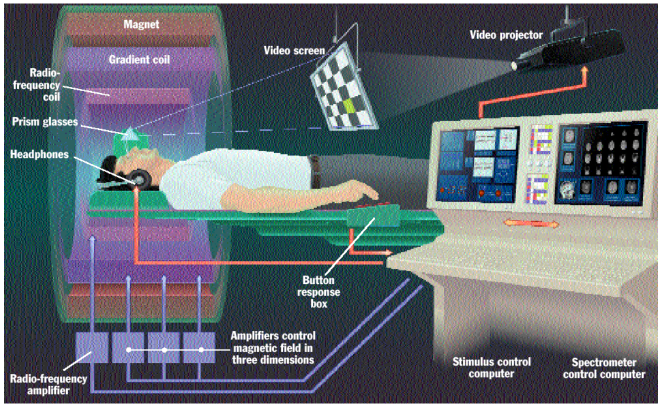
MRI studies brain anatomy.

Functional MRI (fMRI) studies brain function.



http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm

fMRI Setup

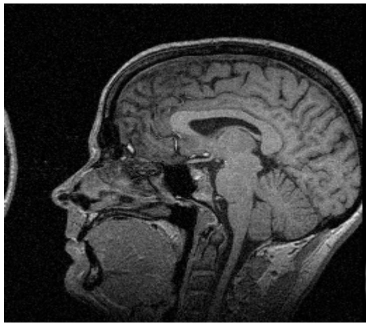


http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm

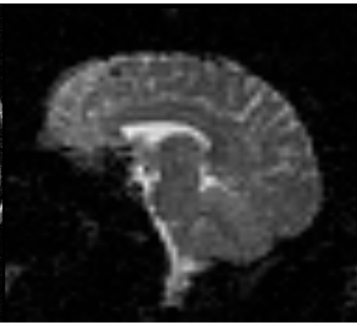
fMRI Acquisition

High spatial resolution

High temporal resolution



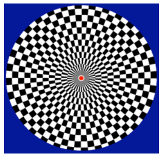
MP-RAGE
Voxel volume: 1 mm^3
Imaging time: 6 min



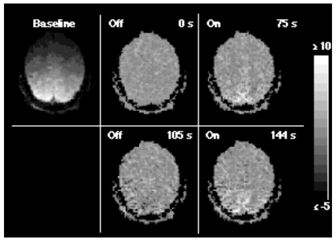
EPI
Voxel volume: 45 mm^3
Imaging time: 60 msec

Buxton 2002

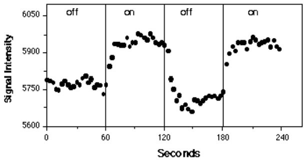
Visual Activation



Flickering Checkerboard
OFF (60 s) - ON (60 s) -OFF (60 s) - ON (60 s) - OFF (60 s)



Brain Activity

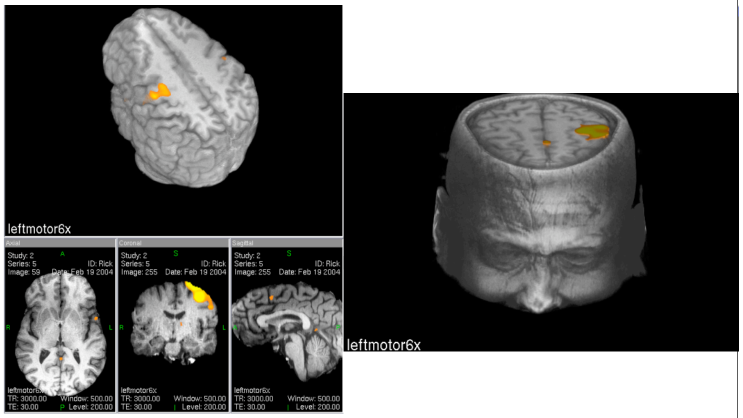


Time ⇨

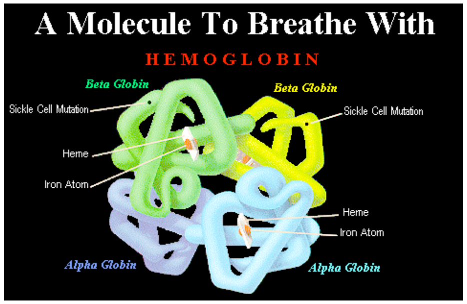
Source: Kwong et al., 1992

http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm

Finger Tapping Task



Hemoglobin



Oxygen binds to the iron atoms to form oxyhemoglobin HbO_2

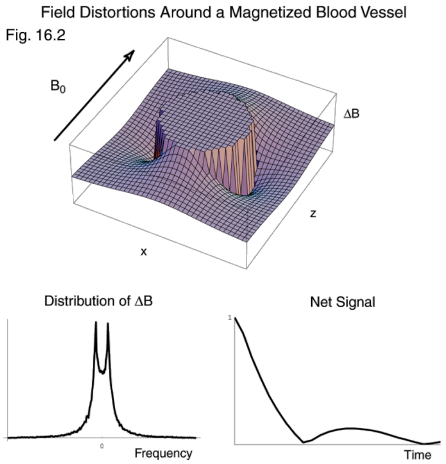
Release of O_2 to tissue results in deoxyhemoglobin $dHBO_2$

http://www.people.virginia.edu/~rjh9u/hemoglob.html

Effect of $dHBO_2$

$dHBO_2$ is paramagnetic due to the iron atoms. As it becomes oxygenated, it becomes less paramagnetic.

$dHBO_2$ perturbs the local magnetic fields. As blood becomes more deoxygenated, the amount of perturbation increases and there is more dephasing of the spins. Thus as $dHBO_2$ increases we find that T_2^* decreases and the amplitude exp(-TE/ T_2^*) image of a T_2^* weighted image will decrease. Conversely as $dHBO_2$ decreases, T_2^* increases and we expect the signal amplitude to go up.

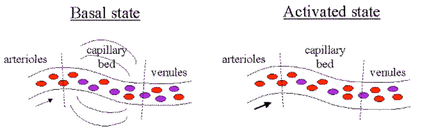


Buxton 2002

BOLD Effect

Blood Oxygen Level Dependent signal

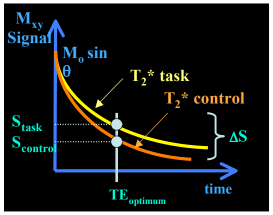
↑neural activity ➔ ↑ blood flow ➔ ↑ oxyhemoglobin ➔ ↑ T2* ➔ ↑ MR signal



- normal flow
- basal level [Hbr]
- basal CBV
- normal MRI signal

● = HbO_2
● = Hbr

- increased flow
- decreased [Hbr] (*lower field gradients around vessels*)
- increased CBV
- increased MRI signal (*from lower field gradients*)

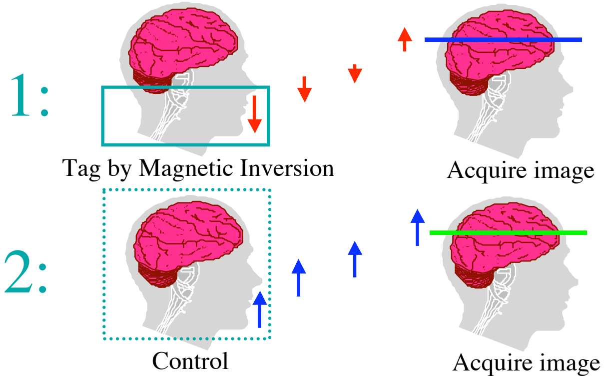


Source: fMRIB Brief Introduction to fMRI

Source: Jorge Jovicich

http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm

Arterial spin labeling (ASL)

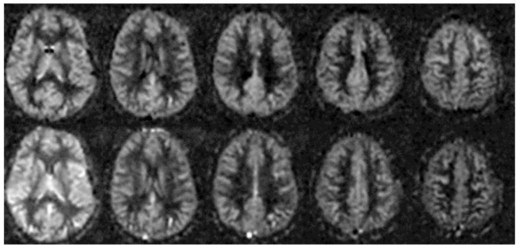


Control - Tag = ΔM ∝ CBF

Multislice CASL and PICORE

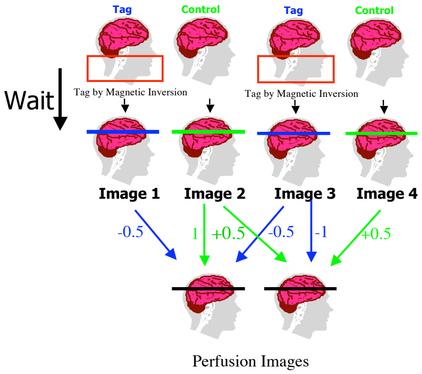
CASL

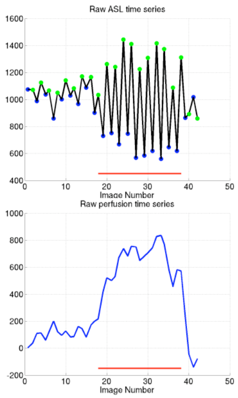
PICORE QUIPSS II



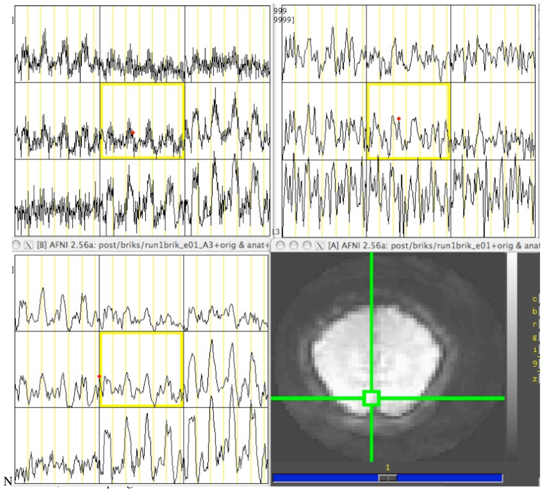
Credit: E. Wong

ASL Time Series

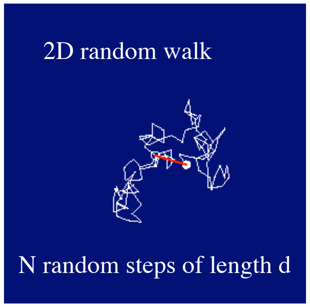




CBF and BOLD Time Series

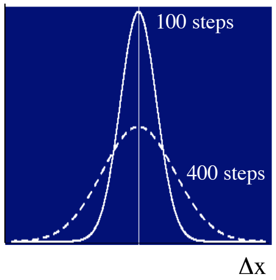


Diffusion



2D random walk

N random steps of length d



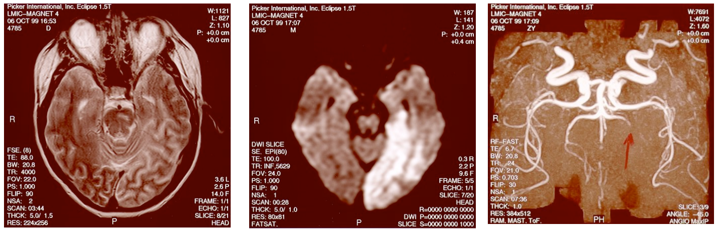
$<\Delta x^2> = Nd^2 = 2DT$

D = diffusivity

In brain:
$D \cong 0.001\ \mathrm{mm}^2/\mathrm{s}$
For T=100 msec,
$\Delta x \cong 15\ \mu$

Credit: Larry Frank

Diffusion Weighted Images



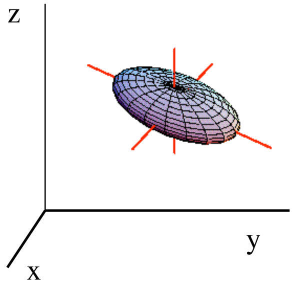
T2 weighted | Diffusion Weighted | Angiogram

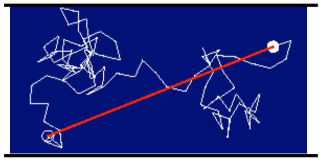
After a stroke, normal water movement is restricted in the region of damage. Diffusivity decreases, so the signal intensity increases.

http://lehighmri.com/cases/dwi/patient-b.html

Restricted Diffusion

D depends on direction

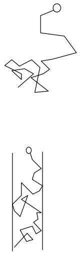


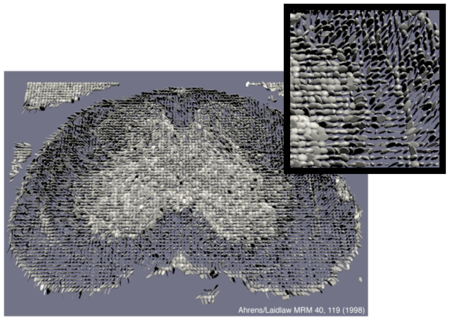


Diffusion tensor:
3 values of D
3 angles

Credit: Larry Frank

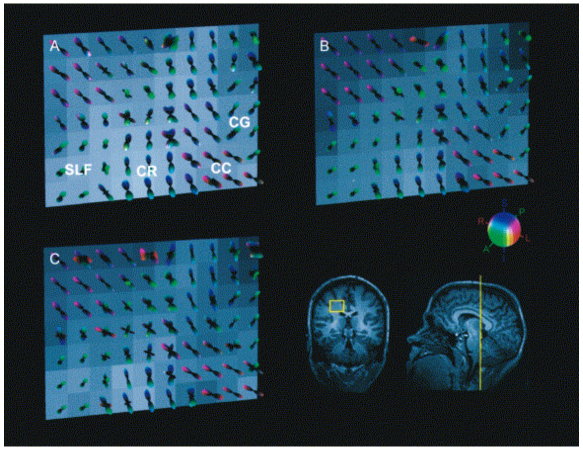
Diffusion Imaging Example





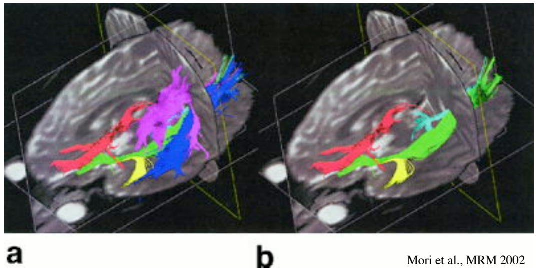
Ahrens/Laidlaw MRM 40, 119 (1998)

Q-ball imaging



Tuch et al, Neuron 2003

Fiber Tract Mapping



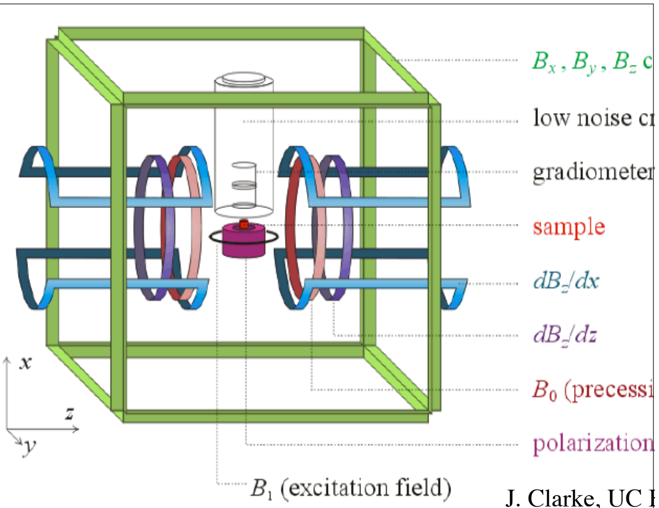
Mori et al., MRM 2002

Timeline

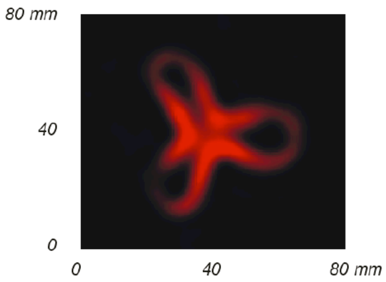
Michael Crichton, 1999

"Most people", Gordon said, "don't realize that the ordinary hospital MRI works by changing the quantum state of atoms in your body ... But the ordinary MRI does this with a very powerful magnetic field - say 1.5 tesla, about twenty-five thousand times as strong as the earth's magnetic field. We don't need that. We use Superconducting QUantum Interference Devices, or SQUIDs, that are so sensitive they can measure resonance just from the earth's magnetic field. We don't have any magnets in there".

J. Clarke, UC Berkeley



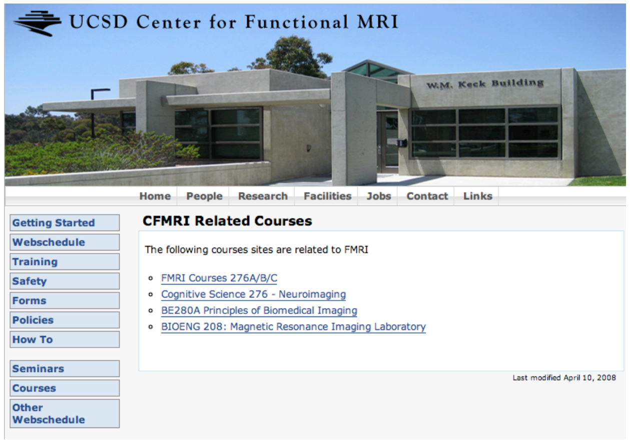
J. Clarke, UC



B0 = 132μT
G = 100μT/m
48 projections
1.5 min. acquisition

J. Clarke, UC Berkeley

Learning More



UCSD Center for Functional MRI

CFMRI Related Courses

The following courses sites are related to FMRI

- FMRI Courses 276A/B/C
- Cognitive Science 276 - Neuroimaging
- BE280A Principles of Biomedical Imaging
- BIOENG 208: Magnetic Resonance Imaging Laboratory

Last modified April 10, 2008