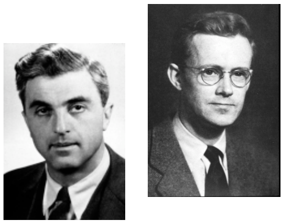
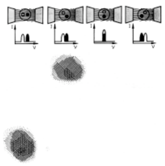
ECE187
Introduction to Biomedical Imaging

Fall Quarter 2004
MRI Lecture

History of MRI



1946: Felix Bloch (Stanford) and Edward Purcell (Harvard) demonstrate nuclear magnetic resonance (NMR)



1973: Paul Lauterbur (SUNY) published first MRI image in Nature.

History of MRI

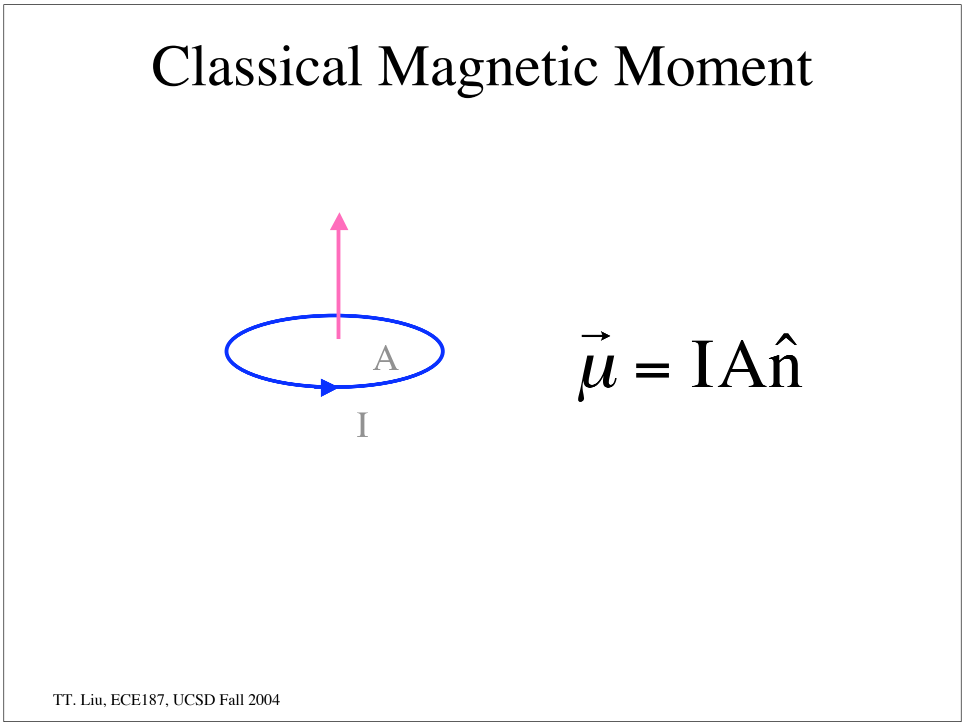
Late 1970's: First human MRI images

Early 1980's: First commercial MRI systems

1993: functional MRI in humans demonstrated

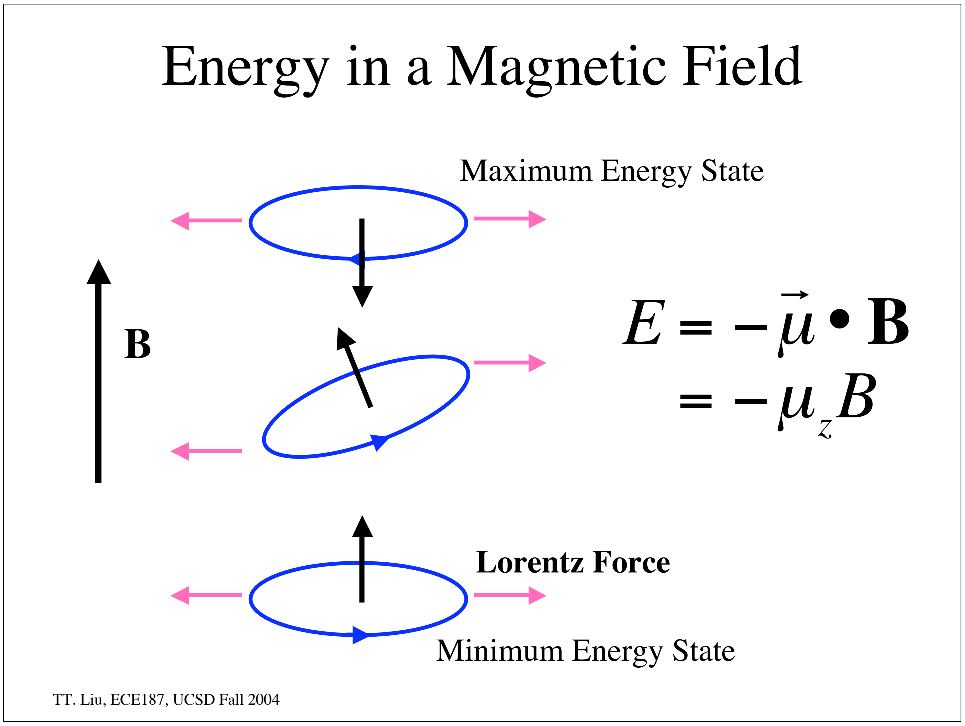
Spin

- Intrinsic angular momentum of elementary particles -- electrons, protons, neutrons.
- Spin is quantized. Key concept in Quantum Mechanics.



Classical Magnetic Moment

$$\vec{\mu} = \mathrm{I}\mathrm{A}\hat{\mathrm{n}}$$

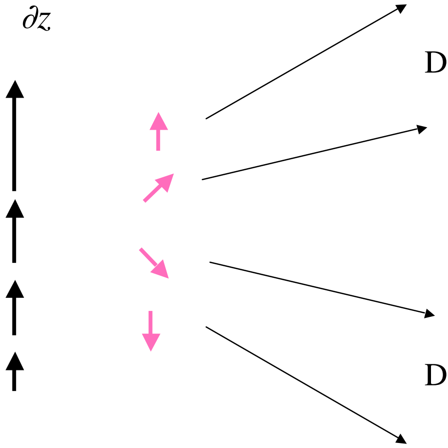


Energy in a Magnetic Field

$$\begin{aligned} E &= -\vec{\mu} \bullet \mathbf{B} \\ &= -\mu_z B \end{aligned}$$

Force in a Field Gradient

$$\mathbf{F} = -\nabla E = \mu_z \frac{\partial B_z}{\partial z}$$

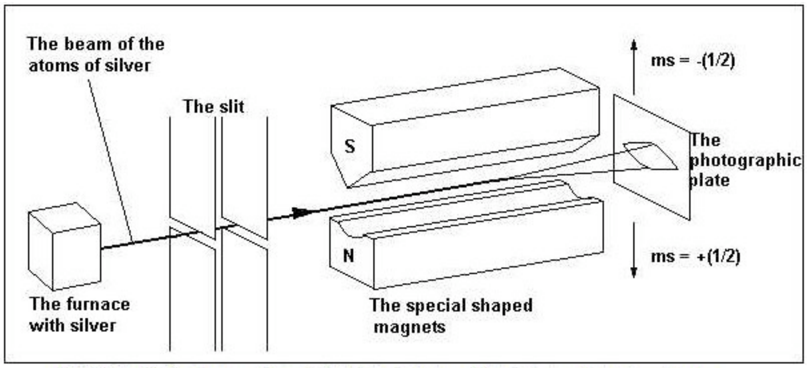


Deflected up

Increasing
vertical
B-field.

Deflected down

Stern-Gerlach Experiment



The Stern-Gerlach experiment. On the photographic plate are two clear tracks.

Image from http://library.thinkquest.org/19662/high/eng/exp-stern-gerlach.html?tqskip=1

Stern-Gerlach Experiment

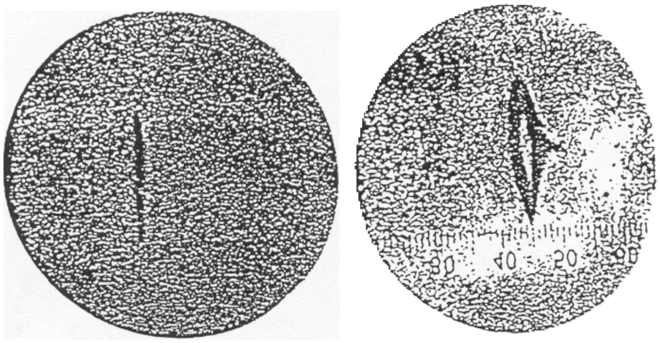


Image from http://library.thinkquest.org/19662/high/eng/exp-stern-gerlach.html?tqskip=1

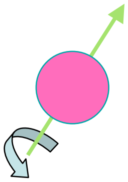
Quantization of Magnetic Moment

The key finding of the Stern-Gerlach experiment is that the magnetic moment is quantized. That is, it can only take on discrete values.

In the experiment, the finding was that

$$\mu_z = +\mu_0 \text{ OR } -\mu_0$$

Magnetic Moment and Angular Momentum



A charged sphere spinning about its axis has angular momentum and a magnetic moment.

This is a classical analogy that is useful for understanding quantum spin, but remember that it is only an analogy!

Relation: $\boldsymbol{\mu} = \gamma\, \mathbf{S}$ where γ is the gyromagnetic ratio and **S** is the spin angular momentum.

Quantization of Angular Momentum

Because the magnetic moment is quantized, so is the angular momentum.

In particular, the z-component of the angular momentum Is quantized as follows:

$$S_z = m_s \hbar$$

$$m_s \in \left\{ -s, -(s-1), \ldots s \right\}$$

s is an integer or half intege

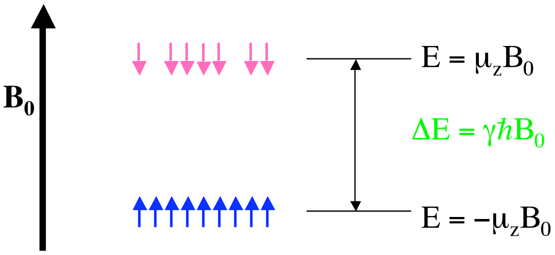
Hydrogen Proton

Spin 1/2

$$S_z = \begin{cases} +\hbar/2 \\ -\hbar/2 \end{cases}$$

$$\mu_z = \begin{cases} +\gamma\hbar/2 \\ -\gamma\hbar/2 \end{cases}$$

Boltzmann Distribution



B_0

$E = \mu_z B_0$

$\Delta E = \gamma\hbar B_0$

$E = -\mu_z B_0$

$$\frac{\text{Number Spins Up}}{\text{Number Spins Down}} = \exp(-\Delta E/kT)$$

Ratio = 0.999990 at 1.5T !!!

Corresponds to an excess of about 10 up spins per million

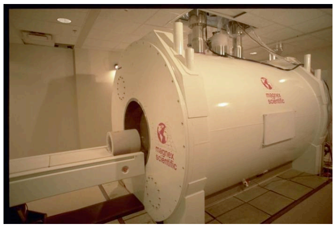
Equilibrium Magnetization

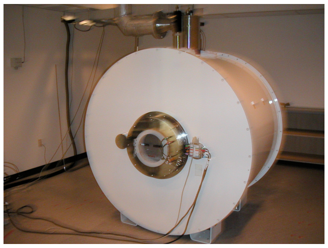
$$\begin{aligned}\mathbf{M}_0 &= N\langle \mu_z \rangle = N\left(\frac{n_{up}(-\mu_z) + n_{down}(\mu_z)}{N}\right)\\ &= N\mu\frac{e^{\mu_z B/kT} - e^{-\mu_z B/kT}}{e^{\mu_z B/kT} + e^{-\mu_z B/kT}}\\ &\approx N\mu_z^2 B/(kT)\\ &= N\gamma^2\hbar^2 B/(4kT)\end{aligned}$$

N = number of nuclear spins per unit volume
Magnetization is proportional to applied field.

Bigger is better

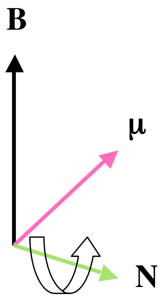

3T Human imager at UCSD.


7T Human imager at U. Minn.


7T Rodent Imager at UCSD


9.4T Human imager at UIC

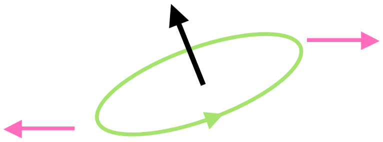
Torque



For a non-spinning magnetic moment, the torque will try to align the moment with magnetic field (e.g. compass needle)

$\mathbf{N} = \mu \times \mathbf{B}$

Torque



Precession

Torque

$$\left.\begin{aligned} \mathbf{N} &= \mu \times \mathbf{B} \\ \frac{d\mathbf{S}}{dt} &= \mathbf{N} \end{aligned}\right\}$$

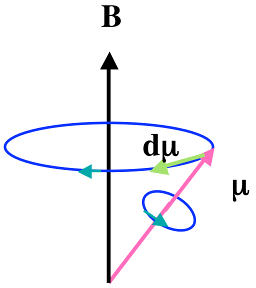
Change in
Angular momentum

$$\left.\begin{aligned} \frac{d\mathbf{S}}{dt} &= \mu \times \mathbf{B} \\ \mu &= \gamma\,\mathbf{S} \end{aligned}\right\} \quad \frac{d\mu}{dt} = \mu \times \gamma\mathbf{B}$$

Relation between
magnetic moment and
angular momentum

Precession

$$\frac{d\mu}{dt} = \mu \times \gamma \mathbf{B}$$



Analogous to motion of a gyroscope

Precesses at an angular frequency of

$$\omega = \gamma \, B$$

This is known as the **Larmor** frequency.

Larmor Frequency

$\omega = \gamma \, B$	Angular frequency in rad/sec
$f = \gamma \, B / (2 \pi)$	Frequency in cycles/sec or Hertz, Abbreviated Hz

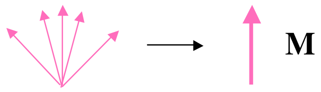
For a 1.5 T system, the Larmor frequency is 63.86 MHz which is 63.86 million cycles per second. For comparison, KPBS-FM transmits at 89.5 MHz.

Note that the earth’s magnetic field is about 50 μT, so that a 1.5T system is about 30,000 times stronger.

Magnetization Vector

$$\mathbf{M} = \frac{1}{V} \sum_{\substack{\text{protons} \\ \text{in V}}} \mu_i$$

Vector sum of the magnetic moments over a volume.



For a sample at equilibrium in a magnetic field, the transverse components of the moments cancel out, so that there is only a longitudinal component.

$$\boxed{\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}}$$

Equation of motion is the same form as for individual moments.

RF Excitation

At equilibrium, net magnetizaion is parallel to the main magnetic field. How do we tip the magnetization away from equilibrium?

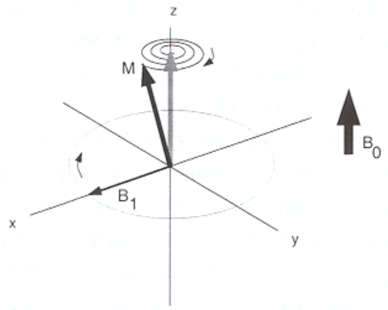
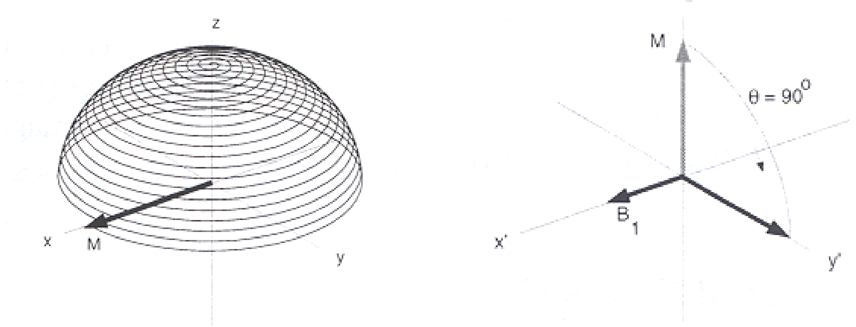


Image & caption: Nishimura, Fig. 3.2

B_1 radiofrequency field tuned to Larmor frequency and applied in transverse (*xy*) plane induces nutation (at Larmor frequency) of magnetization vector as it tips away from the *z*-axis.
- lab frame of reference

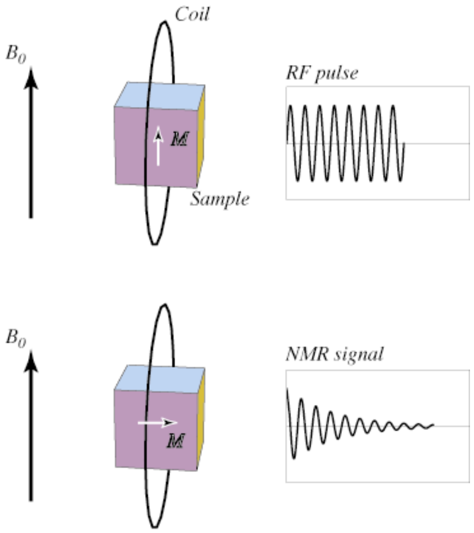
RF Excitation



a) Laboratory frame behavior of **M** b) Rotating frame behavior of **M**

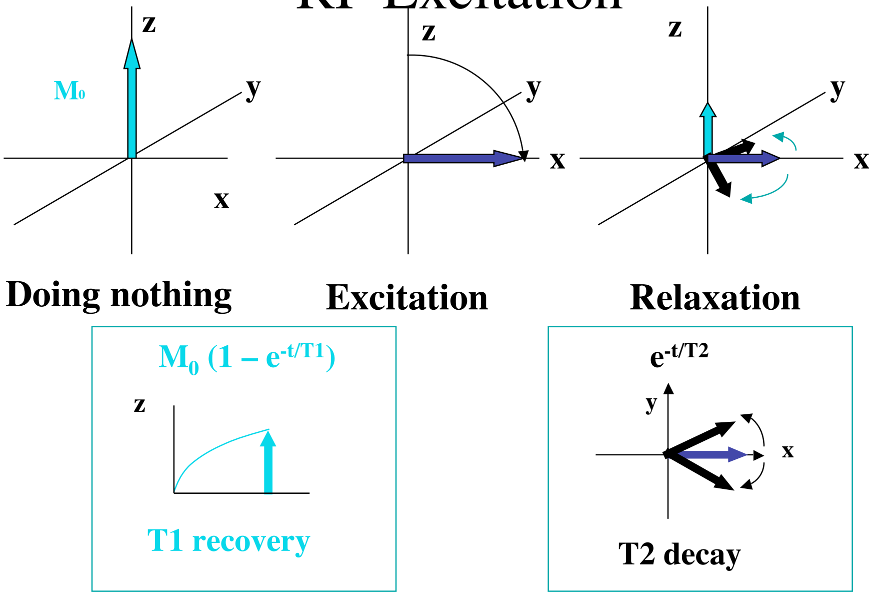
B_1 induces rotation of magnetization towards the transverse plane. Strength and duration of B_1 can be set for a 90 degree rotation, leaving **M** entirely in the *xy* plane. Images & caption: Nishimura, Fig. 3.3

RF Excitation



From Buxton 2002

RF Excitation



Doing nothing **Excitation** **Relaxation**

$M_0\,(1 - e^{-t/T1})$

T1 recovery

$e^{-t/T2}$

T2 decay

Credit: Larry Frank

Relaxation

An excitation pulse rotates the magnetization vector away from its equilibrium state (purely longitudinal). The resulting vector has both longitudinal $\mathbf{M}_z$ and tranverse $\mathbf{M}_{xy}$ components.

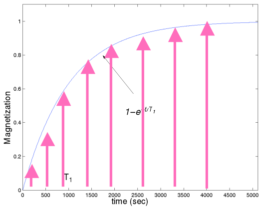
Due to thermal interactions, the magnetization will return to its equilibrium state with characteristic time constants.

T_1 spin-lattice time constant, return to equilibrium of $\mathbf{M}_z$

T_2 spin-spin time constant, return to equilibrium of $\mathbf{M}_{xy}$

Longitudinal Relaxation

$$\frac{d\mathbf{M}_z}{dt} = -\frac{M_z - M_0}{T_1}$$

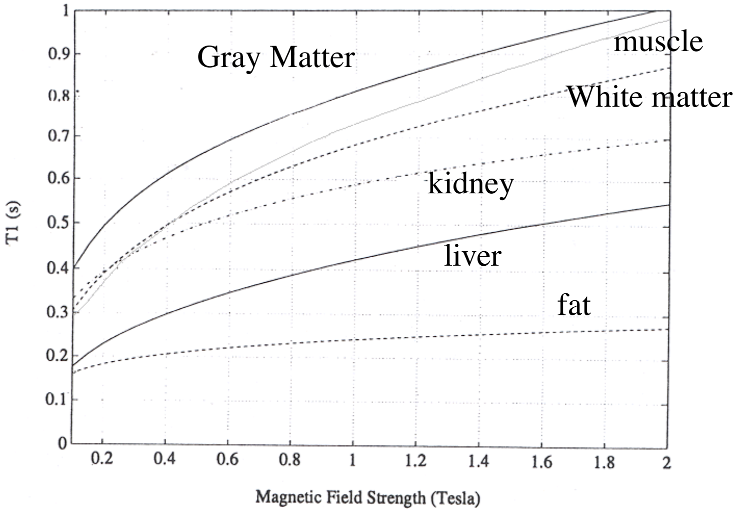


After a 90 degree pulse $$M_z(t) = M_0(1 - e^{-t/T_1})$$

Due to exchange of energy between nuclei and the lattice (thermal vibrations). Process continues until thermal equilibrium as determined by Boltzmann statistics is obtained.

The energy ΔE required for transitions between down to up spins, increases with field strength, so that T_1 increases with **B**.

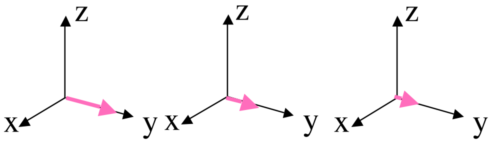
T1 Values



Image, caption: Nishimura, Fig. 4.2

Transverse Relaxation

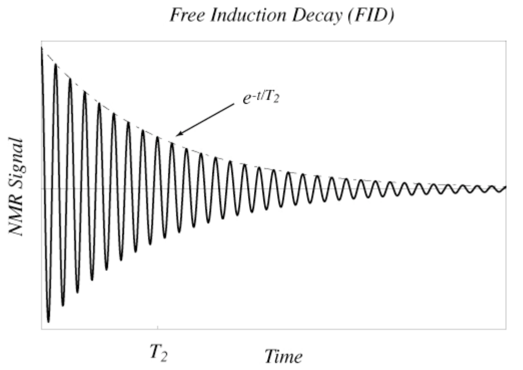
$$\frac{d\mathbf{M}_{xy}}{dt} = -\frac{M_{xy}}{T_2}$$



Each spin's local field is affected by the z-component of the field due to other spins. Thus, the Larmor frequency of each spin will be slightly different. This leads to a dephasing of the transverse magnetization, which is characterized by an exponential decay.

T_2 is largely independent of field. T_2 is short for low frequency fluctuations, such as those associated with slowly tumbling macromolecules.

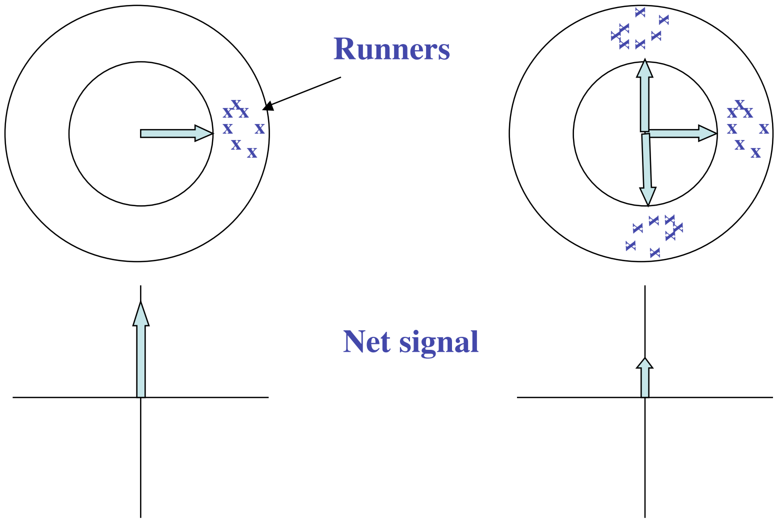
T2 Relaxation



After a 90 degree excitation

$$M_{xy}(t) = M_0 e^{-t/T_2}$$

T2 Relaxation



Credit: Larry Frank

T2 Values

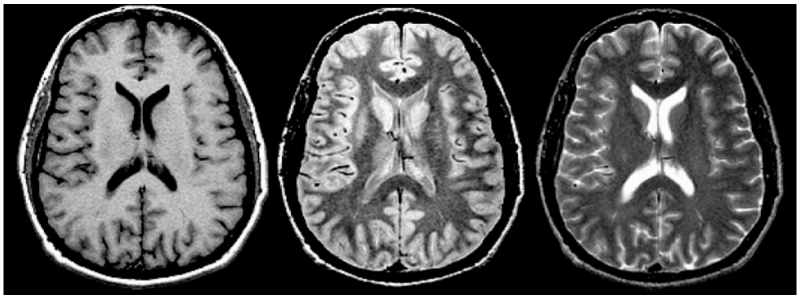
Tissue	T_2 (ms)
gray matter	100
white matter	92
muscle	47
fat	85
kidney	58
liver	43
CSF	4000

Table: adapted from Nishimura, Table 4.2

Solids exhibit very short T_2 relaxation times because there are many low frequency interactions between the immobile spins.

On the other hand, liquids show relatively long T_2 values, because the spins are highly mobile and net fields average out.

Example



T_1-weighted Density-weighted T_2-weighted

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Bloch Equation

$$\frac{d\mathbf{M}}{dt} = \underbrace{\mathbf{M} \times \gamma \mathbf{B}}_{\text{Precession}} - \underbrace{\frac{M_x\mathbf{i} + M_y\mathbf{j}}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_z - M_0)\mathbf{k}}{T_1}}_{\text{Longitudinal Relaxation}}$$

i, j, k are unit vectors in the x,y,z directions.

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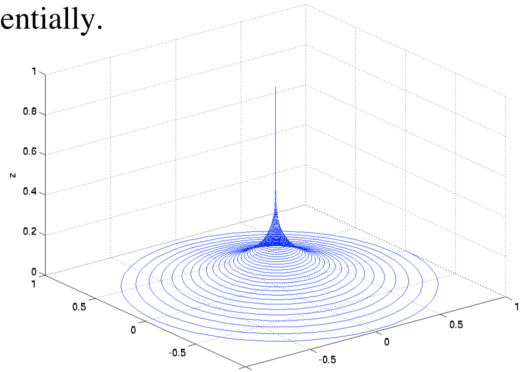
Summary

1) Longitudinal component recovers exponentially.
2) Transverse component precesses and decays exponentially.



Fact: Can show that $T_2 < T_1$ in order for $|M(t)| \le M_0$

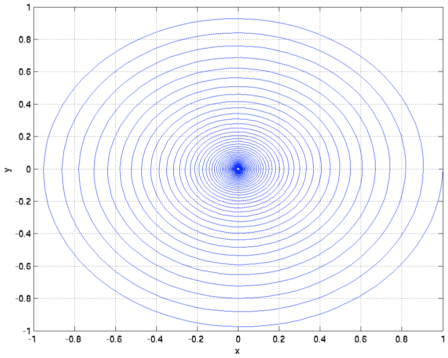
Physically, the mechanisms that give rise to T_1 relaxation also contribute to transverse T_2 relaxation.

Transverse Component

$M \equiv M_x + jM_y$

$$dM/dt = d/dt\left(M_x + iM_y\right)$$
$$= -j\left(\omega_0 + 1/T_2\right)M$$

$$M(t) = M(0)e^{-j\omega_0 t}e^{-t/T_2}$$

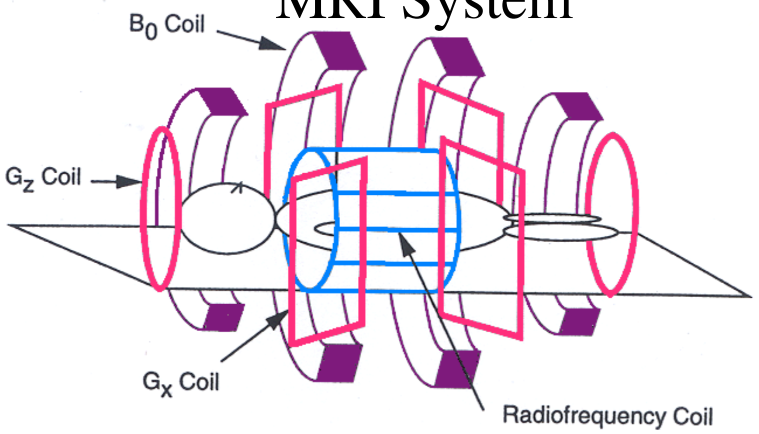


Gradients

Spins precess at the Larmor frequency, which is proportional to the local magnetic field. In a constant magnetic field $B_z=B_0$, all the spins precess at the same frequency (ignoring chemical shift).

Gradient coils are used to add a spatial variation to B_z such that $B_z(x,y,z) = B_0+\Delta B_z(x,y,z)$. Thus, spins at different physical locations will precess at different frequencies.

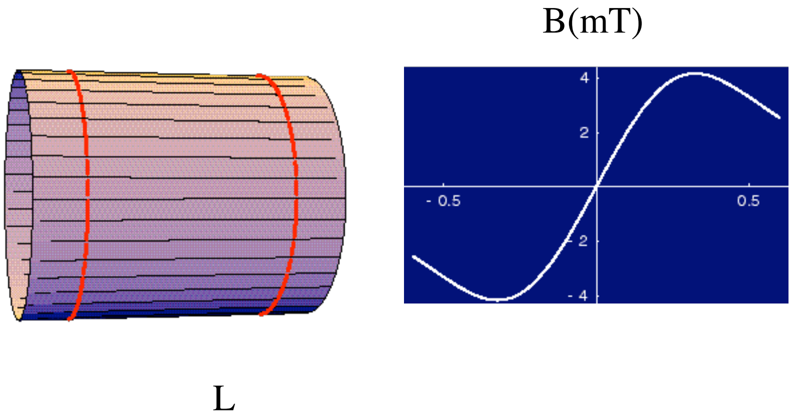
MRI System



Simplified Drawing of Basic Instrumentation.
Body lies on table encompassed by
coils for static field B_o,
gradient fields (two of three shown),
and radiofrequency field B_1.

Image, caption: copyright Nishimura, Fig. 3.15

Z Gradient Coil

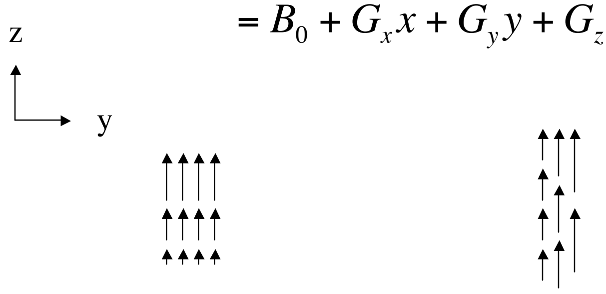


Credit: Buxton 2002

Gradient Fields

$$B_z(x,y,z) = B_0 + \frac{\partial B_z}{\partial x}x + \frac{\partial B_z}{\partial y}y + \frac{\partial B_z}{\partial z}z$$

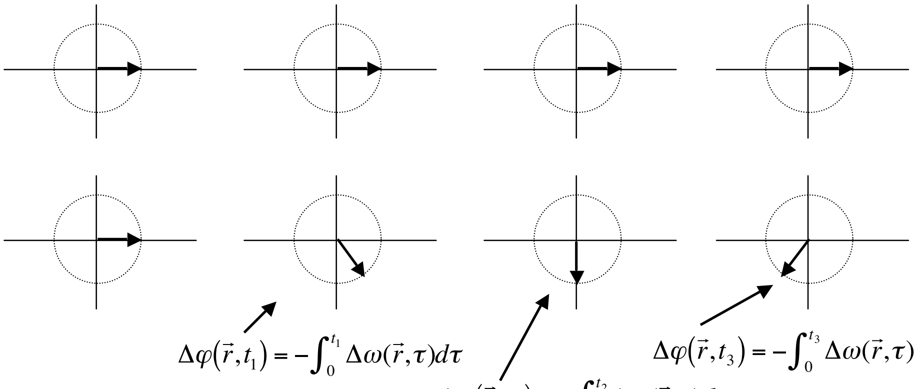
$$= B_0 + G_x x + G_y y + G_z z$$



$$G_z = \frac{\partial B_z}{\partial z} > 0$$

$$G_y = \frac{\partial B_z}{\partial y} > 0$$

Precession with Gradient Field



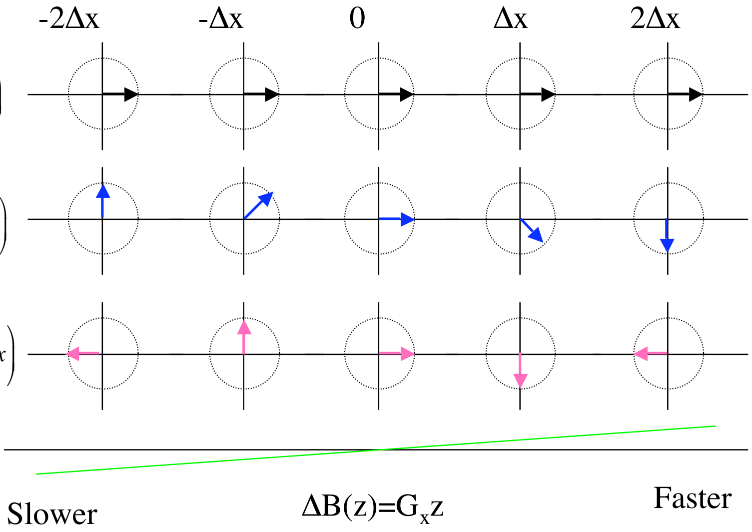
$$\Delta\varphi(\vec{r},t_1) = -\int_0^{t_1} \Delta\omega(\vec{r},\tau)d\tau$$

$$\Delta\varphi(\vec{r},t_2) = -\int_0^{t_2} \Delta\omega(\vec{r},\tau)d\tau$$
$$= -\Delta\omega(\vec{r})t_2$$

if $\Delta\omega$ is non-time varying.

$$\Delta\varphi(\vec{r},t_3) = -\int_0^{t_3} \Delta\omega(\vec{r},\tau)d\tau$$

Interpretation



$$\exp\left(-j2\pi\left(\frac{0}{8\Delta x}\right)x\right)$$

$$\exp\left(-j2\pi\left(\frac{1}{8\Delta x}\right)x\right)$$

$$\exp\left(-j2\pi\left(\frac{2}{8\Delta x}\right)x\right)$$

$\Delta B(z)=G_x z$

K-space

At each point in time, the received signal is the Fourier transform of the object

$$s(t) = M\big(k_x(t), k_y(t)\big) = F\big[m(x,y)\big]\Big|_{k_x(t),k_y(t)}$$

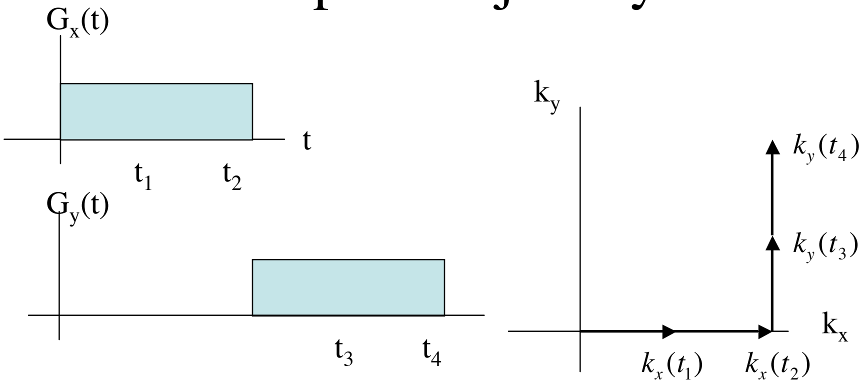
evaluated at the spatial frequencies:

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

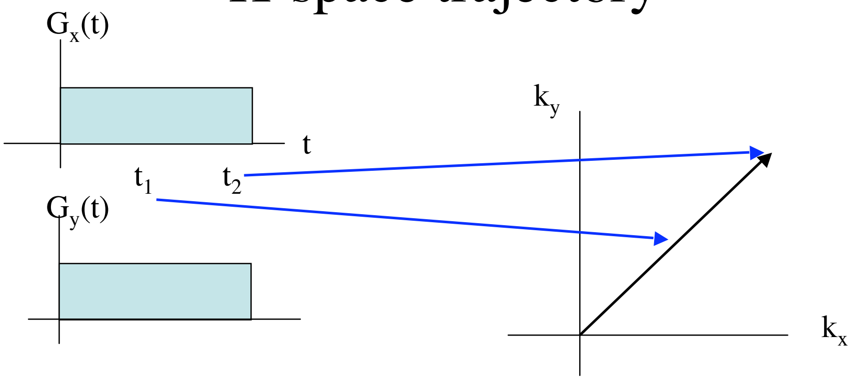
$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

Thus, the gradients control our position in k-space. The design of an MRI pulse sequence requires us to efficiently cover enough of k-space to form our image.

K-space trajectory



K-space trajectory



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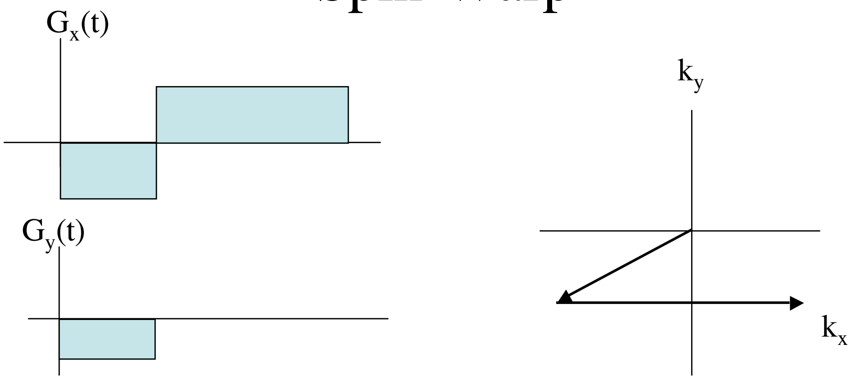
Spin-Warp



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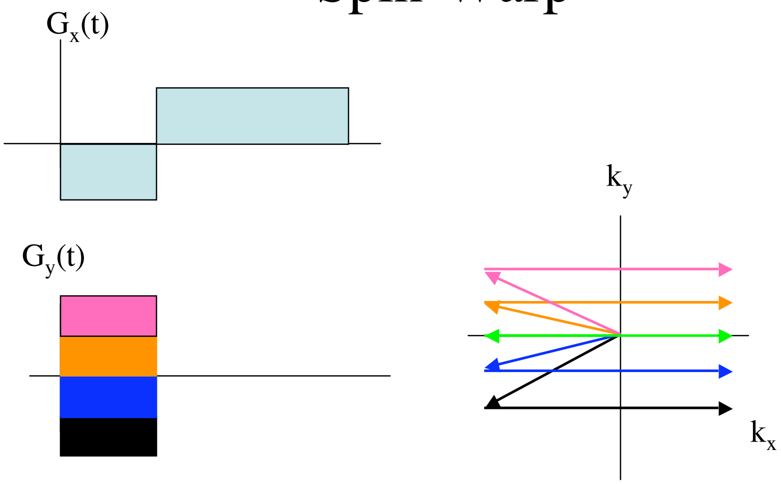
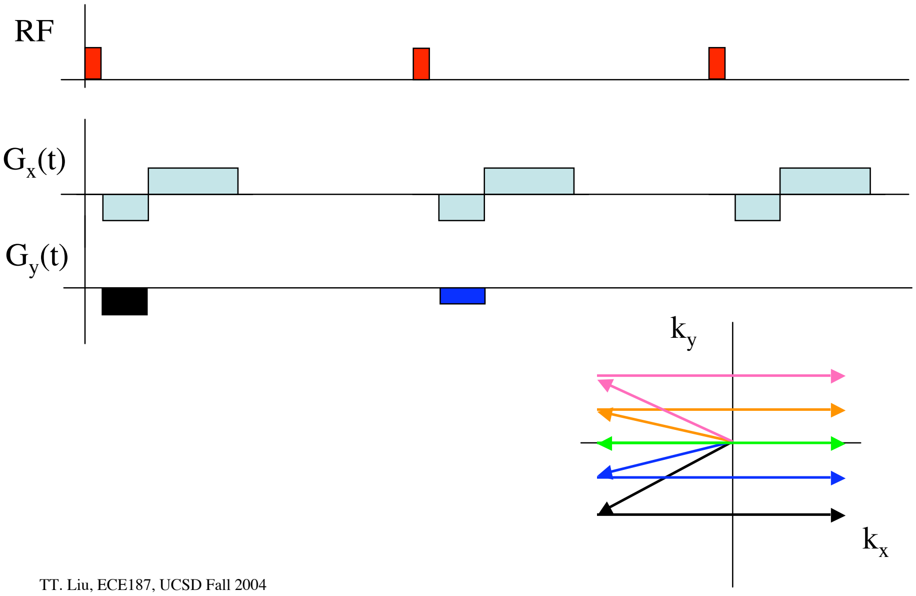
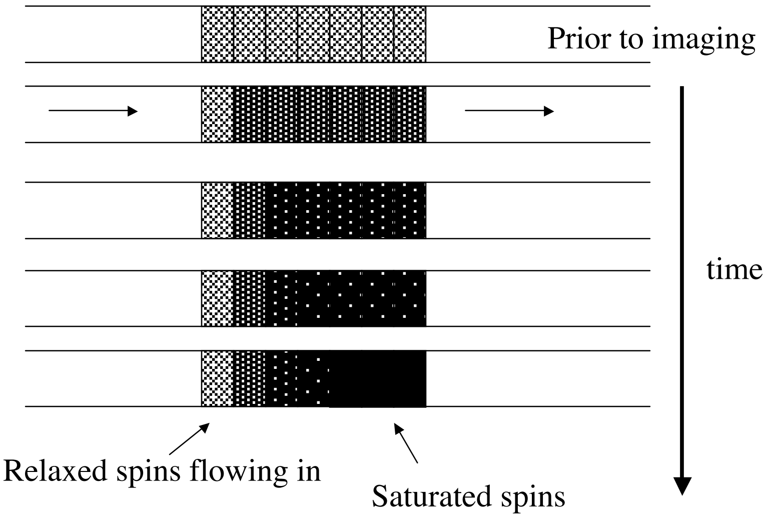
Spin-Warp



Spin-Warp Pulse Sequence



Inflow Effect



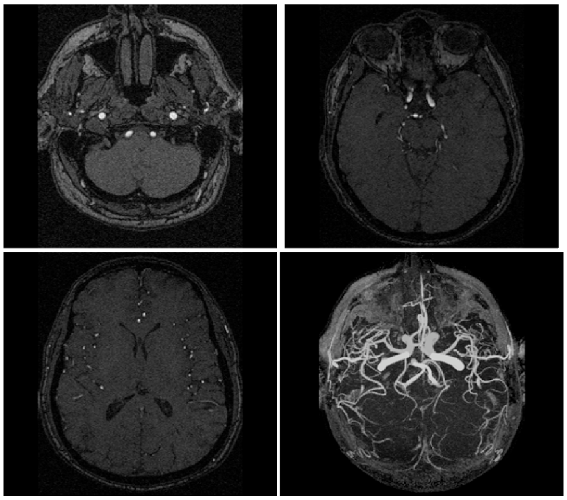
Prior to imaging

time

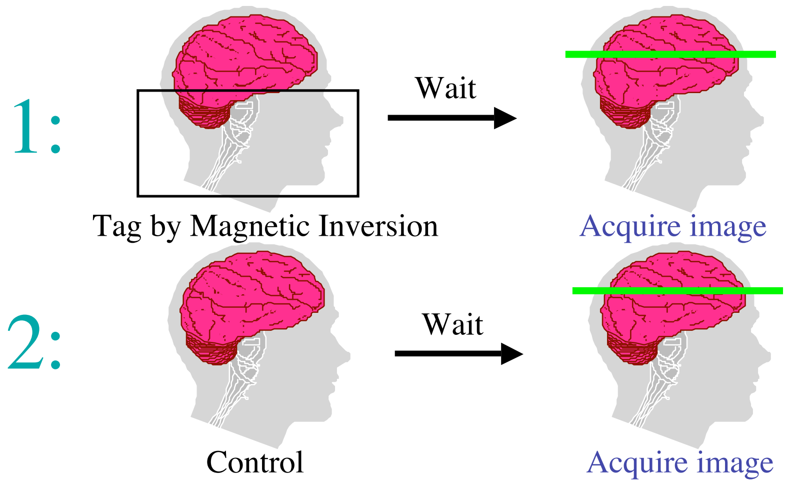
Relaxed spins flowing in

Saturated spins

Time of Flight Angiography



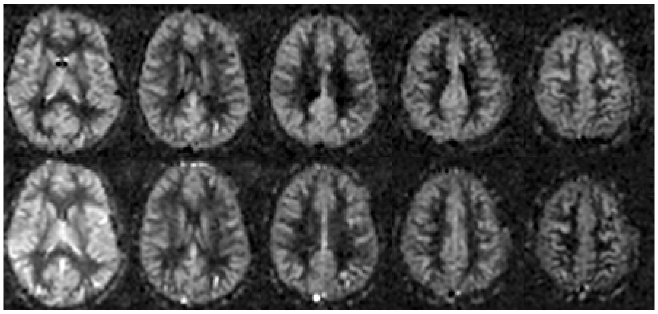
Arterial Spin Labeling (ASL)



Control - Tag $\propto$ CBF

Credit: Wen-Ming Luh

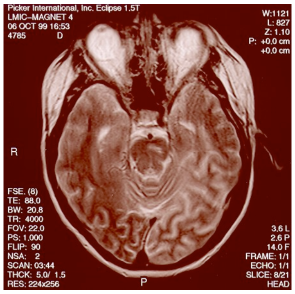
Multislice CASL and PICORE



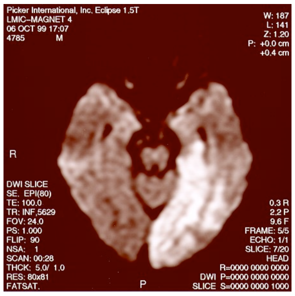
Credit: E. Wong

Diffusion Weighted Images

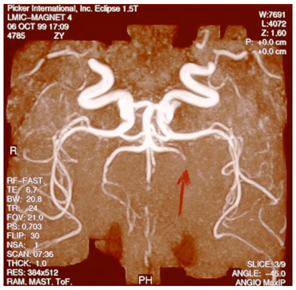
T2 weighted



Diffusion Weighted



Angiogram

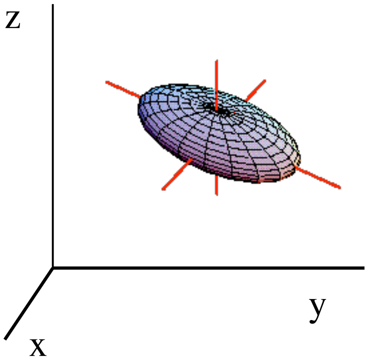


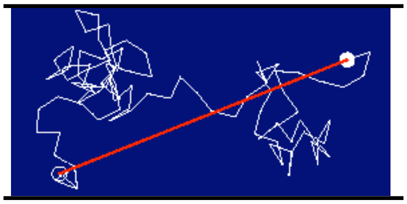
After a stroke, normal water movement is restricted in the region of damage. Diffusivity decreases, so the signal intensity increases.

http://lehighmri.com/cases/dwi/patient-b.html

Restricted Diffusion

D depends on direction

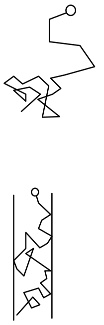


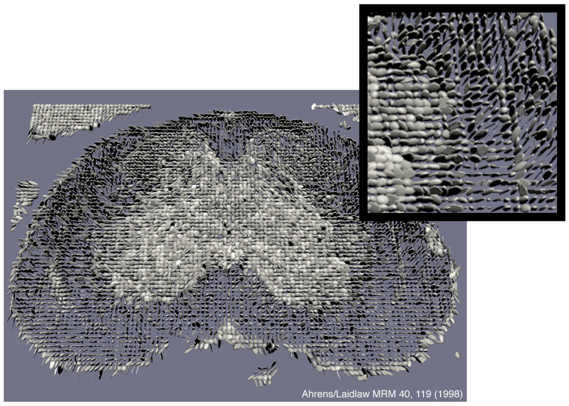


Diffusion tensor:
3 values of D
3 angles

Credit: Larry Frank

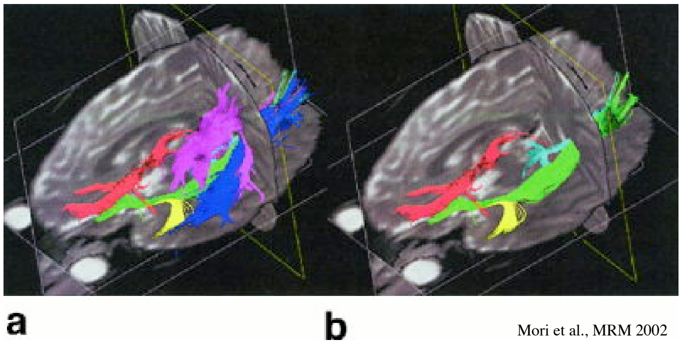
Diffusion Imaging Example





Ahrens/Laidlaw MRM 40, 119 (1998)

Fiber Tract Mapping

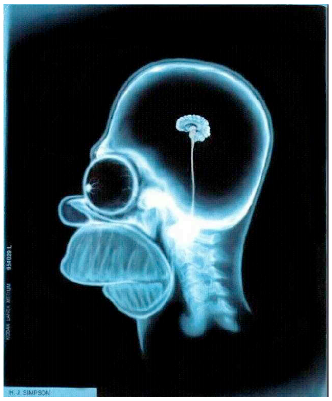


Mori et al., MRM 2002

fMRI

MRI studies brain anatomy.

Functional MRI (fMRI) studies brain function.



http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm

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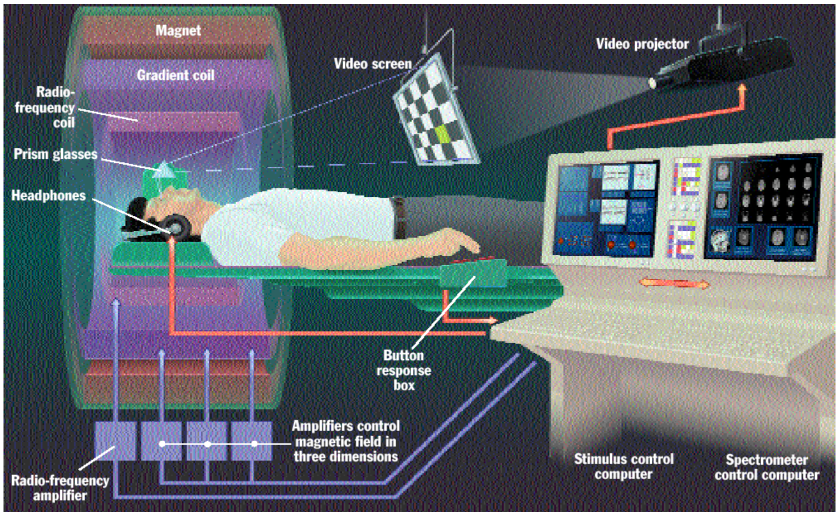
fMRI Setup

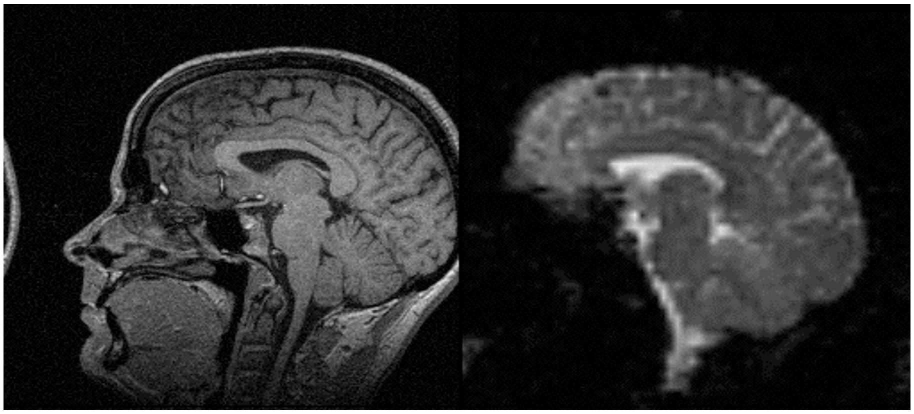


http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm

fMRI Acquisition

High spatial resolution

High temporal resolution

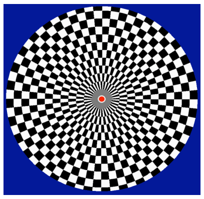


MP-RAGE
Voxel volume: 1 mm^3
Imaging time: 6 min

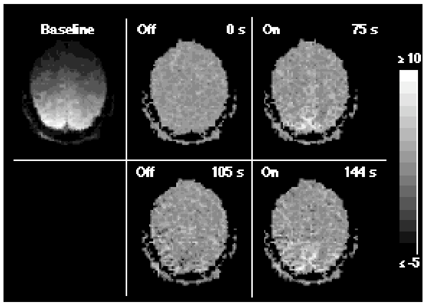
EPI
Voxel volume: 45 mm^3
Imaging time: 60 msec

Buxton 2002

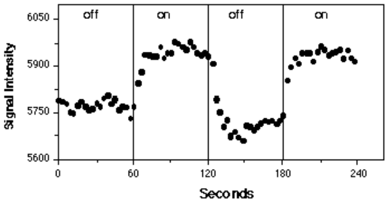
Visual Activation



Flickering Checkerboard
OFF (60 s) - ON (60 s) -OFF (60 s) - ON (60 s) - OFF (60 s)



Brain Activity

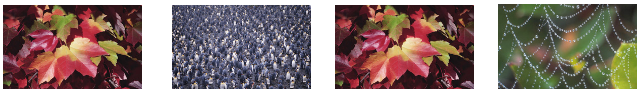


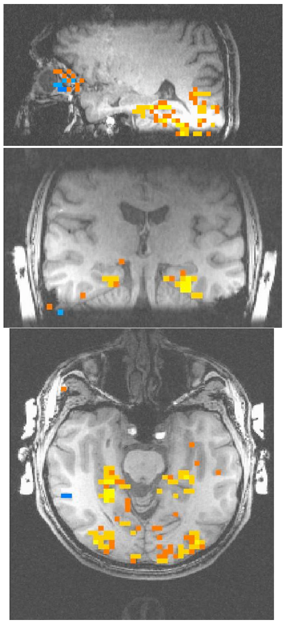
Time ⇨

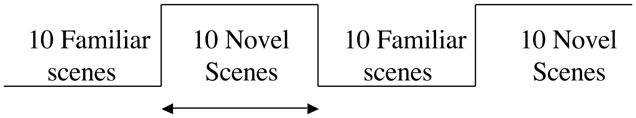
Source: Kwong et al., 1992

http://defiant.ssc.uwo.ca/Jody_web/fmri4dummies.htm

Memory Encoding







10 Familiar scenes

10 Novel Scenes

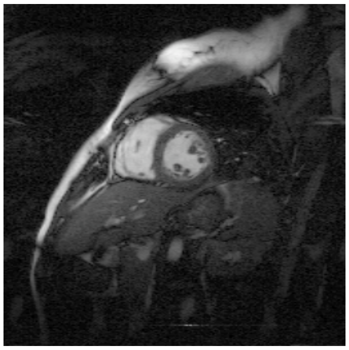
10 Familiar scenes

10 Novel Scenes

25 seconds

The same 4 familiar scenes are used throughout the experiment

Cardiac Imaging



Cardiac Tagging